Atlantic Evaluation and
Research Consultants

K-12 Mathematics Curriculum Review

Final Report

Submitted to the
Department of Education
Newfoundland and Labrador

November, 2007

Atlantic Evaluation and Research Consultants
79 Cherry Lane
Conception Bay South NL
A1W 3B4

Phone (709) 834-5288
Cell (709) 685-1143
Fax (709) 834-538

EXECUTIVE SUMMARY

Purpose and Methods

This review was commissioned by the Department of Education as part of its development of an *Excellence in Mathematics* strategy for the province. This strategy arose out of concerns about the adequacy of the mathematics curriculum in the province. These concerns centre around perceptions that students are expected to achieve too many outcomes, that the program pays inadequate attention to basic mathematical skills and operations, and that the textbooks in some grades are inadequate.

The review had three main purposes: 1) to prepare a summary of research in K-12 mathematics teaching and learning; 2) to provide a comparative analysis of provincial mathematics curriculum in Canada and in other countries (specifically those with a record of high performance on international mathematics achievement studies); and 3) to analyze the strengths and challenges in the current mathematics curriculum in Newfoundland and Labrador, based on the literature review and comparative analysis as well as on consultations with teachers and other stakeholders in the province.

The methods used included a literature review, a comparative analysis of curriculum in other jurisdictions and consultations with major stakeholders. The latter consisted of a series of focus groups and a survey of teachers at selected grade levels.

Main Findings

- The literature seems to support the idea that mathematics should be taught for conceptual understanding and that this does not interfere with basic skill development. However, it is not at all clear that this view enjoys public or teacher support.

- Although there is some support in principle for the conceptual/investigative approach to mathematics, there is strong evidence that teachers feel that the balance has shifted too far and that students are progressing through the grades without having mastered basic skills needed as more advanced mathematics content is introduced. Evidence from parents is more limited but the submissions and focus groups clearly support the teacher view.

- More specifically, there is a strong view on the part of teachers that the mathematics curriculum is too crowded and that it is impossible in most grades to cover all of the expected outcomes in the depth required for adequate student learning.

- A second major issue is a perceived lack of match between outcomes and textbooks and a perception that many of the prescribed texts are of poor quality.

- The comparative curriculum analysis revealed that the total number of outcomes included in the local mathematics curriculum is not very different from the totals found in contemporary programs in other jurisdictions. If the local mathematics curriculum has too many outcomes that flaw is shared by other jurisdictions in Canada.

- However, studies in Western Canada have shown similar concerns to those expressed in this study. A new *Common Curriculum Framework* developed by the Western and Northern Canadian Protocol (WNCP, 2006) contains many fewer outcomes and shifts the balance, especially in the early grades, to the numeracy strand. This framework will form the basis for curriculum revision in the four Western provinces and in the territories over the next few years,

- The revised version of the *NCTM Principles and Standards* addresses part of the problem of balance. The *Curriculum Focal Points* document published in 2006 makes it clearer that some areas have higher priority than others and more explicitly identifies priorities in basic skill areas. The new WNCP curriculum framework though predating the *Focal Points,* is consistent with this direction.

- The combination of a crowded curriculum and diverse student needs and abilities seems to make the working lives of many teachers highly complex and stressful. While this is not confined to mathematics, we were given the distinct impression that mathematics is one significant source of this complexity. Unfortunately, we were unable to explore this phenomenon in the detail needed to draw strong conclusions about teacher workload and its impact on learning.

- The available comparative achievement evidence shows that students in this province perform above the international average but exhibit slight and persistent lower performance relative to the Canadian average and to the highest-performing provinces. On local measures, most of the evidence points to improvement in recent years. However, it is not possible to attribute any of these results directly to the curriculum. Many other factors are known to have small cumulative effects on achievement and curriculum is not likely to be the decisive factor.

- There was concern that teachers have not received adequate professional development to deal with the current curriculum. Taken further, the view was expressed by some that primary/elementary teachers are often not comfortable with mathematics. This was supported to some extent by the survey data. It is not clear if this problem can be solved by professional development. However, it is clear that more intensive PD efforts are required if the principles underlying the new mathematics program are to be maintained. This is especially so if further significant revisions are made.

- There is strong support for program differentiation at the high school level but there are varying views on its extent and on how far down the grade levels this should extend. We heard calls for having a choice of courses in mathematics as early as Grade 5 and other more cautionary views for not closing doors to mathematics too early. A majority of high school teachers feel that the three-level course structure should be extended downward to Level I (Grade 10).

- Many teachers are highly frustrated with having to deal with ISSP development and implementation and, more generally, with the disruption of normal classroom activity caused by the presence of high-needs students or those with severe behaviour problems. Few are critical of the inclusion approach in principle but feel that the system is insufficiently resourced to be successful. Investigating this issues was not part of the mandate of this study. The issue is raised because it illustrates how one aspect of school and classroom activities can impact on others.

Conclusions

- The concerns raised by the stakeholder groups in our consultations are severe enough to warrant the conclusion that the mathematics curriculum is not sustainable in its current form and that minor changes will be insufficient to solve the problem. The concerns over too many outcomes and the quality of textbooks appear well founded. The argument that basic skills are being neglected is more difficult to sustain but likely is related to the crowded nature of the curriculum and the inclusion of content in all of the mathematics strands at all grade levels.

- The current program is based on the original 1989 edition of the *Standards* published by the National Council of Teachers of Mathematics. A new edition, called *Principles and Standards* has been available since 2000. A further document, the *Curriculum Focal Points*, issued in 2006 has clarified the *Principles and Standards* and established content priorities for the various strands at different grade levels. This document makes it clear that all strands do not have equal priority at any one grade level.

- Other jurisdictions, particularly those under the Western and Northern Canadian Protocol (WNCP) which encompasses the four western provinces and the three territories have moved to develop a "second generation" mathematics curriculum, based on the 2000 NCTM *Principles and Standards* and consistent with the *Curriculum Focal Points. Common Curriculum Frameworks* have been developed for Grades K-9 (2006) and Grades 10-12 (2007).

- Our main conclusion is that, rather than embarking on a new curriculum development program that would take several years to reach the implementation stage, it is more appropriate to adopt the WNCP *Common Curriculum Framework*s as the basis for the next generation mathematics curriculum in this province.

- It is important to note that the need for curriculum revision is driven by concerns that the program is too crowded and by the need to move towards a new generation program based on the most recent version of the *Principles and Standards* and the *Curriculum Focal.* It is not driven by any strong evidence that performance of our students has deteriorated. Nevertheless, despite some evidence of an improving trend on local measures, the small but persistent gap in performance relative to the highest performing provinces is a source of concern. Adopting a curriculum that is the same as found in high performing provinces such as Alberta and British Columbia will remove one source of this gap and perhaps allow a focus on other possible sources.

- In light of this achievement gap and recognizing government's commitment to improvement, as evidenced by the *Excellence in Mathematics Strategy,* this is an appropriate time to reiterate the goal of creating a culture of high achievement and to restate the more specific target of bringing the achievement of students in this province to a level comparable to the best in Canada. The immediate target for this is mathematics, although the goal is applicable across the whole curriculum.

- Professional development of teachers in support of curriculum change is a major concern. There is little guidance in the literature on best practices in professional development, other than that it should be more sustained than the typical one-day workshop. Some options for delivery of professional development are discussed in the report and a role for the newly appointed numeracy support teachers in this work is proposed.

SUMMARY OF RECOMMENDATIONS

1. That the WNCP Common Curriculum Frameworks for Mathematics K-9 and Mathematics 10-12 (WNCP, 2006 and 2007) be adopted as the basis for the K-12 mathematics curriculum in this province.

2. That implementation commence with Grades K, 1, 4, 7 in September, 2008, followed by in Grades 2,5,8 in 2009 and Grades 3,6,9 in September 2010.

3. That the senior high school program be implemented on the same schedule as now proposed for the Western and Northern jurisdictions, starting in 2010.

4. That the proposed revised program not be piloted but that an effort be made to learn from the initial experiences in other jurisdictions implementing the program in 2007-08.

5. That textbooks and other resources specifically designed to match the WNCP frameworks be adopted as an integral part of the proposed program change.

6. That implementation of the proposed changes to the mathematics curriculum be accompanied by an introductory professional development program designed to introduce the curriculum to all mathematics teachers at the appropriate grade levels prior to the first year of implementation.

7. That at least partial support for professional development be negotiated with publishers as part of a textbook adoption package.

8. That numeracy support teachers have a primary role in delivery of PD for primary/elementary teachers.

9. That the responsibilities of mathematics department heads in intermediate and high schools (whether 7-12, 10-12 or any other combination) include facilitating introductory PD sessions and follow-up of these sessions.

10. That, in the short term, numeracy support teachers assigned to Grades K-6 be considered as lead teachers for mathematics in the schools for which they are responsible.

11. That the work of numeracy support teachers be systematically monitored for at least two years, using methods designed to assess their impact on fidelity of implementation and on outcomes.

12. That, following this period, a determination be made of whether this program should be continued or whether the resources would be better utilized to support lead teachers at the individual school level.

13. That the Department of Education reinforce, through reference in its curriculum documents and professional development activities, to well established features of effective teaching: maximizing the use of time, maximizing student engagement in academically meaningful work, high expectations, maximizing content coverage, monitoring and using assessment to improve learning.

14. That the Department of Education reinforce the value of homework, establish guidelines on the amount and type of homework to be assigned, especially in the early grades, and develop and disseminate a parent guide to homework.

15. That any revisions to the model for providing services to special needs students include provision for remedial work for those students requiring additional time to meet grade level expectations.

TABLE OF CONTENTS

EXECUTIVE SUMMARY I

SUMMARY OF RECOMMENDATIONS V

LIST OF TABLES AND FIGURES IX

LIST OF ABBREVIATIONS X

I INTRODUCTION 1

- PURPOSE 1
- THE PROBLEM 1
- BACKGROUND 2
- RESEARCH QUESTIONS AND REQUIRED ANALYSIS 4
- LITERATURE REVIEW 5
- COMPARATIVE CURRICULUM ANALYSIS 5
- CONSULTATIONS 6
- TEACHER SURVEY 7

II ISSUES IN MATHEMATICS CURRICULUM, TEACHING AND LEARNING 8

- MATHEMATICS AS A SCHOOL SUBJECT 8
- THEORETICAL PERSPECTIVES 10
- ASSESSMENT AND ACCOUNTABILITY 12
- MATHEMATICS CURRICULUM 13
- THE WNCP RESEARCH STUDY 19
- FACTORS INFLUENCING TEACHING AND LEARNING IN MATHEMATICS 21
- THE SAIP AND PISA MATHEMATICS STUDIES 25
- TEACHER PROFESSIONAL DEVELOPMENT 29
- WHAT DOES THE RESEARCH TELL US ABOUT MATHEMATICS CURRICULUM, TEACHING AND LEARNING? 31

III COMPARATIVE CURRICULUM ANALYSIS 33

- K-9 COMPARISONS 34
- HIGH SCHOOL PROGRAMS 37
- TEXTBOOKS AND OTHER RESOURCES 40

IV CONSULTATIONS 45

- SUBMISSIONS 45
- FOCUS GROUPS 46
- TEACHER SURVEY 53

V MATHEMATICS PERFORMANCE OF STUDENTS IN NEWFOUNDLAND AND LABRADOR 60

NATIONAL AND INTERNATIONAL COMPARISONS (SAIP AND PISA) 61
UNIVERSITY MATHEMATICS COURSES 64
CRITERION REFERENCED TEST RESULTS 70

VI SUMMARY, ANALYSIS AND RECOMMENDATIONS 72

SUMMARY 72
CURRICULUM DEVELOPMENT 75
IMPLEMENTATION SCHEDULE 79
TEXTBOOKS 81
STUDENT ACHIEVEMENT 83
PROFESSIONAL DEVELOPMENT 85
INSTRUCTIONAL PRACTICES 91

REFERENCES 95

LIST OF TABLES AND FIGURES

Table

2.1 Factors Associated with Mathematics Achievement:
SAIP Mathematics Assessment, 2001 27
3.1 Number of Outcomes by Program and Grade 35
3.2 Degree of Match of Outcomes by Program and Grade 37
3.3 Comparison of High School Course Structures 39
3.4 Comparison of Textbooks and Other Resources 43

Figure

5.1 SAIP Mathematics: 13-Year-Olds 63
5.2 SAIP Mathematics: 16-Year-Olds 64
5.3 PISA Mathematics 64
5.4 Mean Score Mathematics 1000 66
5.5 Percent Pass Mathematics 1000 66
5.6 Mean Score Mathematics 1090 67
5.7 Percent Pass Mathematics 1090 67
5.8 Mean Scores and Pass Rates for Mathematics 3205: 2001-2007 69
5.9 Mean Scores and Pass Rates for Mathematics 3204: 2001-2007 69
5.10 Mean Score on Mathematics Criterion Referenced Tests 72

LIST OF ABBREVIATIONS

APEF	Atlantic Provinces Education Foundation
CAMET	Council of Atlantic Ministers of Education and Training (successor to APEF)
CMEC	Council of Ministers of Education Canada
CRT(s)	Criterion Referenced Tests (Newfoundland and Labrador)
GCO(s)	General Curriculum Outcomes
KSCOs	Key Stage Curriculum Outcomes
NCTM	National Council of Teachers of Mathematics
OECD	Organization for Economic Cooperation and Development
PISA	Programme for International Student Assessment
SAIP	School Achievement Indicators Program
SCO(s)	Specific Curriculum Outcomes
TIMSS	Trends in International Mathematics and Science Study
WCP	Western Canadian Protocol (from 1995)
WNCP	Western and Northern Canadian Protocol (from 2006)

I INTRODUCTION

Purpose

This review was commissioned by the Department of Education as part of its development of an *Excellence in Mathematics* strategy for the province. The review had three main purposes: 1) to prepare a summary of research in K-12 mathematics teaching and learning; 2) to provide a comparative analysis of provincial mathematics curriculum in Canada and in other countries (specifically those with a record of high performance on international mathematics achievement studies); and 3) to analyze the strengths and challenges in the current mathematics curriculum in Newfoundland and Labrador, based on the literature review and comparative analysis as well as on consultations with teachers and other stakeholders in the province.

The Problem

The Department of Education's *Excellence in Mathematics* Strategy arose out of concerns about the adequacy of the mathematics curriculum and particularly with performance on provincial, national and international assessments and preparation for higher education. This issue has been the subject of some media attention in the past several years. This seems to have intensified in the past few months, with many letters to the editor, editorials, op-ed articles and attention on local radio current affairs and call-in shows. In November, 2007 alone, as this report was being prepared, we were able to document ten such items in the local media. A few of these appear to have been precipitated by the fact that the study itself is under way. However, most are related to the activities of critics of the program and responses to these critics.

The focal point of many of these interventions has been a perception that the changes implemented since1999, and particularly the adoption of an Atlantic Provinces framework and content, has led to reduced rigour in the mathematics curriculum. Added to this are concerns expressed by parents and teachers that the mathematics curriculum is too crowded, with more expected outcomes than can be covered within the school year. There is also a perception that the shift in emphasis from basic skills to conceptual understanding means that mathematics is now being taught in such a way that parents are unable to help their children with the increased amount and different type of homework that the new curriculum seems to demand.

It is important to note that questions about the performance of students in mathematics are not new. Indeed, a major review of mathematics and science programs was conducted in 1989 (Government of Newfoundland and Labrador, 1989). That review was precipitated mainly by high failure rates in mathematics at the first year university level. The focus of that review was therefore primarily on the high school to post-secondary transition. Although transition issues are not as direct a focus in the current situation, the perception remains that many students are ill-prepared for post-secondary mathematics.

The intense focus on school mathematics programs is also not uniquely local. National media often carry stories about mathematics learning (e.g. Farran, November 19, 2007). In the United States the debate has been so intense as to have been labelled "the math wars" (Schoenfeld, 2004; Marshall, 2006). Some of the background and sources of this debate are examined in the literature review.

Background

The current Mathematics curriculum was implemented between 1999 and 2005. Following a framework and set of standards originally developed by the National Council of Teachers of Mathematics in the United States (NCTM, 1989) and widely adopted in Canada, the curriculum contains four content strands:

- Number Concepts/Number and Relationship Operations
    - demonstrating number sense and applying number-theory concepts; and demonstrating operation sense and applying operation principles and procedure in both numeric and algebraic situations.

- Patterns and Relations
    - exploring, recognizing, representing, and applying patterns and relationships.

- Shape and Space
    - demonstrating an understanding of and applying concepts and skills associated with measurement, and demonstrating spatial sense and applying geometric concepts, properties, and relationships.

- Data Management and Probability
    - solving problems involving the collection, display, and analysis of data and representing and solving problems involving uncertainty.

In the local curriculum, each of these strands has been divided into two sub-strands. These are expressed as a total of eight *General Curriculum Outcomes (GCOs)* that students are expected to meet. All of these outcomes receive at least some attention at all grade levels, in the form of a set of *Specific Curriculum Outcomes (SCOs)* presumably developed at a grade or age appropriate level. This *Spiral Approach* to the curriculum is characteristic of the 1989 NCTM Standards and has been the source of some of the criticism of these Standards and the curriculums derived from them.

The current curriculum is supported by textual and other teacher and student resource materials at each grade level. All of this is available at the Department of Education website (http://www.ed.gov.nl.ca/edu/sp/mathlist.htm). Although published as provincial curriculum materials, these documents have their origins in the work of the Atlantic Provinces Education Foundation (APEF; since superseded by the Council of Atlantic Ministers of Education and Training, CAMET). The guiding structure for the mathematics curriculum is found in the 1989 version of the Curriculum and Evaluation Standards of the National Council of Teachers of Mathematics (NCTM, 1989).

The available documents are based on a broader curriculum framework, which includes statements of outcomes at increasing levels of detail, from *Essential Graduation Learnings* (broad cross-curricular goals) through to *Specific Curriculum Outcomes* embodying the detailed content in each of the above strands at each grade level. Statements of a similar nature were available for other jurisdictions, which greatly facilitated the comparative analysis.

Research Questions and Required Analysis

The Terms of Reference for the review gave three main areas for analysis and a series of research questions within each of these areas.

- A summary of research in K-12 mathematics teaching and learning to include but not limited to the following questions:
  - What approach or approaches to teaching mathematics works best for students at various grade levels (K-12)?
  - What mathematics curriculum design works best for students at various grade levels (K-12)?

- A comparative analysis of the provincial mathematics curriculum across Canada and in other specified countries to include, but not be limited to, the following questions:
  - How does the mathematics curriculum in Newfoundland and Labrador compare to that of other Canadian jurisdictions (e.g., Ontario, Western and Northern Canadian Protocol (WNCP), etc.)?
  - How does the mathematics curriculum in Newfoundland and Labrador compare to that of other international jurisdictions, specifically to those jurisdictions which perform well on international testing such as PISA? Such jurisdictions include Hong Kong-China, Finland, Korea, and Netherlands.

- An analysis of strengths and challenges of the current mathematics curriculum in Newfoundland and Labrador
  - This analysis would come from, (i) the aforementioned research, (ii) the aforementioned jurisdictional comparative analysis, and (iii) consultations with teachers and other stakeholders.

The intended culmination of this research would be recommendations designed to strengthen the mathematics curriculum in Newfoundland and Labrador.

It is important to note that this was intended to be mainly a comparative curriculum review and not an analysis of other sources of potential difficulties in mathematics teaching and learning. Nevertheless, as a way of complementing the comparative analysis, a teacher survey was proposed. This was designed to obtain a more accurate representation of teacher perceptions of the provincial curriculum than could be expected from focus group participants.

Literature Review

The Terms of Reference called for a literature review, specifically focused on identifying best practices in mathematics curriculum, teaching and learning. Fortunately, one of the members of the research team was immersed in the mathematics education literature as part of her doctoral studies, which facilitated this work considerably. Because the volume of literature was large, the review attempted to focus on systematic reviews or syntheses rather than individual research studies. A recent comprehensive review conducted by the Western and Northern Canadian Protocol (WNCP) as part of their review of mathematics curriculum proved to be a highly valuable source because of its currency and because it covered almost identical issues to those of interest here.

A large literature exists around the 1989 NCTM Standards and their impact on curriculum. The interpretation and use of these standards has triggered much of the controversy in mathematics teaching. A new edition, the *Principles and Standards,* published in 2000 is widely (though not necessarily accurately) thought of as having shifted emphasis back to basic skills. Because the current Newfoundland and Labrador curriculum appears to have been built around the 1989 Standards, the distinction between the 1989 and 2000 versions is of some interest, most obviously because one must ask whether the time has come for a revision of the local curriculum based on the 2000 Principles and Standards.

Comparative Curriculum Analysis

This was the core activity for this project, and the one which occupied the most time. This work was divided into three stages as follows:

- Locating the relevant documents
- Initial classifications by staff
- Expert review and comparative judgment on content, depth of coverage, time allocations and other features of the curriculum

The primary documents for the curriculum review were the curriculum guides or similar documents produced by each jurisdiction (or by the consortium jurisdictions). After reviewing several analytical schemes, a simple process was decided on, in which the Newfoundland and Labrador outcomes were placed in the first column of a spreadsheet and the comparable statements from other jurisdictions in further columns across the sheet. Non-matching outcomes were added at the bottom of the appropriate columns to make it easy to identify these elements. Blank cells indicated outcomes included in the local program but not in other jurisdictions. In some cases, this was a matter of different grade level placement and in others it identified local outcomes not found anywhere in other programs.

The initial comparative judgments were made by the researchers. Once a working version was produced, this was subjected to an external panel of mathematics teachers for corroboration.

The above process worked well for Grades K-9 but could not be used at the senior high school level because of the large number of courses and because courses are not organized directly around outcome statements. Instead, a two-stage process was followed. The first stage consisted of a comparison of course levels and structures. The second stage involved a comparison of main topics in all of the local courses with the topics covered in courses designed for comparable students in other jurisdictions.

Consultations

The final major research activity consisted of stakeholder consultations. Focus group sessions were held in the four Anglophone school districts. These involved teachers at each school level, district officials and parent representatives. Focus group sessions were also held in St. John's with representatives of the public post-secondary institutions and provincial representatives of the Newfoundland and Labrador Teachers Association, the School Boards Federation and the School Councils Federation. A call for submissions was also issued, allowing other groups, as well as the general public an opportunity to present views.

Teacher Survey

Because it is difficult to reach large number of teachers through focus groups or other direct contacts, a decision was made to conduct a teacher survey. Survey instruments were designed to solicit teacher perceptions of their preparation to teach mathematics, their teaching assignments and detailed aspects of the curriculum, including ratings of the key stage outcomes (KSCOs) for Grades 3, 6, 9 and 12. The target populations were all teachers of Grades 3, 6 and 9, as well as all senior high school teachers. Four separate instruments were developed, differing mainly in the specific details of curriculum at the various grade levels.

II ISSUES IN MATHEMATICS CURRICULUM, TEACHING AND LEARNING

The literature on mathematics education is extensive. Theoretical debate can be found on the nature of mathematics and mathematics curriculum and theories of learning as applied to mathematics. Research on teaching and learning strategies in mathematics overlaps an even broader generic literature on teaching and learning. It is not possible in a policy review with a short time frame and limited mandate to go back to the hundreds of original studies and attempt to synthesize what is known and not known. Instead, we have relied mainly on existing reviews and syntheses, along with reviews previously conducted by the researchers for other purposes. Many of the individual studies cited in the synthesis literature are familiar to the researchers from other work but we have not gone back to all of the original sources in this case.

The account given here draws heavily from a report prepared in support of the mathematics curriculum review conducted under the Western and Northern Canada Protocol (WNCP). The account is partly a conventional literature review but also attempts to examine the main issues in mathematics curriculum, including mathematics teaching and learning, in terms that we hope will make sense to both policy-makers and the general public.

Mathematics as a School Subject

A review of the report of the 1989 Provincial Task Force on Mathematics and Science Education (Crocker,1989) shows that teaching and learning in mathematics was no less controversial then as now. While the focus has shifted somewhat from the high school/post-secondary transition to difficulties in the earlier grades, student struggles with mathematics are well documented. Indeed, while speaking favourably about the then new 1989 NCTM *Standards*, and recommending that they be adopted as the basis for future mathematics curriculum development in the province, that report noted that any new programs designed to conform to these standards would likely be controversial (p. 40).

That report also argued that there are features of mathematics which contribute to its perceived difficulty as a school subject. Learning mathematics may be likened to learning a complex second language, with little opportunity for practice of that language outside of school. This is especially true as one progresses from basic counting, measurement and arithmetic operations to the more abstract concepts involved in algebraic expressions, equations, analytic geometry or trigonometry. It follows that mathematics is fundamentally a school subject. Children do not encounter mathematics in anything but a rudimentary way anywhere except in school. As the Task Force Report noted, “there is no mathematical equivalent of the bedtime story (p. 133).” Children do not interact with their parents and others in mathematical language in the way they do with normal language. Finally, mathematics is more precise in its formulation and less forgiving of error than most other school subjects. It is possible to communicate in ordinary language using a variety of expressions, dialects or accents, even if these are not very precise. This is not possible in mathematics.

Despite these limitations in experience, children are expected to develop a facility with mathematics that exceeds what is expected in any other school subject except their first language. Mathematics is not an optional subject, even at the end of the high school years, and failure to take mathematics at relatively advanced levels is known to close many doors to higher education and careers. This is clearly understood in schools, where the demand for mathematics courses far exceeds the minimum requirements for graduation from high school. Indeed, students graduating with only the minimum four credits required in mathematics would find themselves effectively barred from most forms of post-secondary education. Almost all students take more than the minimum number of mathematics courses. Those who do poorly in mathematics find themselves limited in their post-secondary options because at least first year university mathematics is prerequisite for admission to most scientific, technical or professional schools.

This combination of high demand and limited opportunity to learn mathematics in anything other than a school setting makes it almost inevitable that many students will struggle with mathematics and that the subject will remain one of the more controversial school subjects. This also makes it more compelling that we attempt to optimize curriculum and instructional practices to ensure that students can do as well as possible and that most can meet the high demands of this subject.

Theoretical Perspectives

Early theoretical approaches to mathematics teaching and learning were derived from the contrasting psychological theories of behaviourism and Gestalt theory. The first required that learning tasks be broken into small steps, with each step learned through a process of conditioning through practice and reinforcement. Steps could eventually be accumulated into a larger body of learning. The second approach involved a search for patterns, relationships and applications from which it was assumed that a deeper understanding would emerge.

Although these perspectives have been superseded by more elaborated theories, the fundamental contrast between learning through practice and learning through a search for meaning persists, and remains the basis for much of the controversy in mathematics teaching and learning. This is evidenced by the continual tension between those who advocate a “back to basics” approach to mathematics curriculum and those who advocate teaching the underlying meaning for mathematical concepts and operations. Related to this is the tension between rote and discovery learning and between teaching as transmission of knowledge and teaching as facilitating learning. These tensions are found not only among professionals in the field but are evident in the public debate we are witnessing locally, nationally and internationally.

A major shift of emphasis occurred in the 1960s and 1970s, as part of a larger reform movement in mathematics and science. The “new mathematics”[1] of that era was characterized more by epistemological (theories of knowledge) than by psychological theories. This era was characterized by the involvement of the scientific community in both mathematics and science curriculum development, with attempts to define the “structure of the discipline” becoming the dominant approach. This was supported by the developmental psychology of Piaget and Dienes, as promoted by prominent educators such as Jerome Bruner.[2] Bruner’s famous dictum that anyone can learn anything at any level provided it is presented in a developmentally appropriate way is the basis for the “spiral curriculum” that is the subject of much of today’s controversy in mathematics curriculum. In this respect, the 1960s reform movement remains with us today.

[1] Interestingly, the term “new mathematics” is widely used by parents to describe the current curriculum, implying that this is dramatically different from what they learned in school. This may reflect a “back to basics” approach found in the 1980s when many of today’s parents attended school. An earlier generation, those attending school in the 1960s and 1970s were almost certainly exposed to curriculum that was radically different from the traditional approach

In the 1980s, epistemology and developmental psychology were superseded (or perhaps absorbed) by the successors of Gestalt theory, cognitive psychology and constructivism. The basis of these approaches is that individuals construct meaning from their experiences and that learning is best approached by problem-solving. This again highlighted the controversy over expository versus discovery approaches to learning that had existed in the 1960s and that also persists to the present. Problem-solving as an approach to teaching and learning is seen by its advocates as a requirement for developing meaning and by its critics as detracting from the drill and practice required for children to master basic mathematical skills.

The most recent era in mathematics education may be traced to the publication in 1989 of the first edition of the *Curriculum and Evaluation Standards for School Mathematics* by the National Council of Teachers of Mathematics (*the Standards*). The *Standards* had two goals: to "create a coherent vision of what it means to be mathematically literate… and a set of standards to guide the revision of the school mathematics curriculum" (NCTM, 1989, p. 1). The *Standards* are organized around the following goals for all students: "(1) learn to value mathematics, (2) become confident in the ability to do mathematics, (3) become mathematical problem solvers, (4) learn to communicate mathematically, and (5) learn to reason mathematically" (NCTM, 1989, p. 5).

The *Standards* thus combined an epistemological emphasis, in identifying a structure for mathematics curriculum, with the thrust towards development of meaning through problem-solving. It is fair to say that *the Standards* has become the defining influence on mathematics curriculum and teaching throughout North America.[3] The influence of that document is clearly evident in the "strands" and the outcomes statements found in the local curriculum, which are almost direct copies of those in *the Standards*, and in the emphasis on "explaining" concepts and operations, rather than on practice of basic skills. Again, this is the source of much of the controversy over the current curriculum.

---

of earlier years.

[2] Bruner's short book, *The Process of Education* was arguably the most influential educational document of the 1960s.

[3] Although the *Standards* have been the subject of much commentary in the literature worldwide, it is not clear if they have been the driving force for mathematics curriculum in other countries and particularly in high achieving non-English speaking countries such as Finland, Korea or Japan.

We might argue from this that the more things change the more they remain the same. The early contrast between behaviourism and Gestalt theory survives in the conflicts between emphasis on skill versus conceptual development in curriculum design and between drill and practice and problem-solving as approaches to teaching and learning. Politically, this conflict seems to play out in the contrast between programs and approaches advocated by professional bodies and the public perception that these approaches are leading to decline in student capabilities in the basic skills that appear to be so highly valued in the larger society.

Assessment and Accountability

Overlaying all of the controversy over the fundamentals of mathematics teaching and learning is the emphasis since the 1990s on accountability for the large public investment being made in education. This emphasis is being driven by the virtually universal agreement that education is becoming more and more important for both individual and societal development and that ever higher levels of educational attainment and achievement are necessary for individuals to function in an increasingly competitive knowledge-based economy. The economic value of education to individuals has been well established and it is possible to extend individual benefits to benefits for the society as whole. Combined with this is increased concern that the education system is not delivering optimum value on the public investment and a perception, at least on the part of some segments of society, that the quality of education is deteriorating rather than improving.

These concerns have resulted in a trend in almost all jurisdictions towards policies designed to increase accountability. In Canada, provincial student testing programs are the most obvious example of this trend. More broadly, the development of national and international assessments, such as the School Achievement Indicators Program (SAIP) in Canada and the Programme for International Student Assessment (PISA) by the countries of the Organization for Economic cooperation and Development (OECD), are part of a larger set of policies and programs designed to report to the public on “indicators” of how well the education system is doing. Publication of the pan-Canadian Education Indicators reports by Statistics Canada and other similar reports are the tangible results of these policies. While it is not clear to what extent such reports reach the general public, they are the obvious result of attempts at public accountability.

Locally, except for a small gap in the 1990s, a public examination system has always been an important component of certification of high school graduates. More recently a system of "criterion referenced tests" (CRTs) has been developed, designed to show how our children are doing in school. These are now administered annually to all students in Grades 3, 6 and 9. By all accounts, the results on the CRTs are taken quite seriously by schools and teachers and scores on these tests are taken as indicators of school as well as individual student performance.

Nevertheless, it is much less obvious that the existence of these tests is contributing to improved achievement. Measuring and reporting on the status quo may be a necessary condition but it is certainly not a sufficient condition for improvement. Improvement requires change. While change may be engendered by high level policies affecting structures, curriculum, resources and the like, which are under the control of governments, none of these policies is of any use unless something happens at the level of the teacher and the student.

Mathematics Curriculum

The primary focus of this study is on the curriculum because curriculum is one of the main features of education that can be determined by central jurisdictional authorities. The results of the comparative curriculum analysis are presented in the next chapter. The emphasis here is on the structure of the mathematics curriculum and its recent evolution.

Typically, since the 1960s, curriculum development begins with some concept of how the discipline is structured. This is especially true in mathematics and science where the content is well defined. For close to twenty years now, the NCTM *Standards* appear to have been the defining authority for mathematics curriculum developers. With the appearance of a revision, the *Principles and Standards* in 2000, this trend appears to be continuing.

The 1989 *Standards*, on which most contemporary curriculum documents are based, organized mathematics under several main content strands, which differ by level. For Grades K-4, the strands are:

- Number
- Operations and Computation

- Geometry and Measurement
- Probability and Statistics
- Patterns and Relationships

Problem-solving is treated as a separate “process” strand intended to cut across all areas. It can be inferred from the document that problem-solving was considered by the *Standards* developers the key aspect of the proposed curriculum changes. Mathematics as communication, mathematics as reasoning, and mathematical connections are also considered as process stands.

These strands also appear at higher levels but with further differentiation. For example, algebra is introduced explicitly for Grades 4-9 and trigonometry and synthetic and algebraic geometry, along with the conceptual underpinnings of calculus, appear in Grades 9-12. The process strands are found throughout. More explicit standards, which resemble the “outcome statements” found in most contemporary mathematics curriculum documents, constitute the specific standards which students are expected to meet at these three key stages.

With respect to instructional practices, the *Standards* are quite clear in advocating increased emphasis on problem-solving as a generic strategy, greater use of questioning techniques, greater student interaction, greater use of computers and calculators and assessment for improving learning. Decreased emphasis is called for on drill and practice, memorization, teaching as transmission and assessment for assigning grades.

Not surprisingly, the publication of the *Standards* intensified the debate over teaching basic skills versus teaching for meaning and problem-solving. Schoenfeld (2003) detailed the political background and evolution of the “math wars” in the United States following the 1983 Reagan administration report *A Nation at Risk*. Although the NCTM Standards were developed in response to the documented low achievement of American students, this was insufficient to satisfy critics of *the Standards*. To the advocates of basic skills, this document served to entrench an approach to teaching which was widely blamed for low achievement.

In Canada, the political reaction to *the Standards* and the broader controversies in mathematics education has been more muted. The reasons for this are not entirely clear. However, Canadian students have tended to perform much better than their American counterparts on international assessments and the tendency to place a strict interpretation on provincial responsibility for education means that there are few national organizations which might lead the debate on such matters. To be sure, at a local level, many of the same issues, especially around basic skills versus interpretation and meaning, are often heard and sometimes lead to major reviews such as the 1989 Task Force study in this province and smaller initiatives such as the current study.

Despite the controversy, *the Standards* quickly became the primary reference point for curriculum development in both the United States and Canada. Accompanying this trend was another significant shift in curriculum development with the formation of regional groupings in both the Western and the Atlantic provinces. These were driven by a perception of duplication of effort, and also perhaps by recognition that core curriculum areas, particularly mathematics and science, have little local variation and that the existence of professionally developed standards facilitates a collaborative approach to curriculum development among provinces.[4] Both the APEF curriculum used locally and the curriculum based on the Western Canadian (now Western and Northern Canadian) Protocol grew out of these initiatives.

It is a measure of how long an innovation takes to gain a foothold that by the time the 1989 *Standards* were fully integrated into curriculum the NCTM, having been hard at work in the 1990s, issued a new edition (NCTM, 2000). Seven years later, we are now seeing the 2000 edition appearing as the basis for curriculum development in other jurisdictions in Canada (specifically Ontario in 2005 and the Western and Northern Jurisdictions in 2006).

The 2000 document, now titled *Principles and Standards for School Mathematics* begins with a set of six principles:

- Equity
    - Excellence in mathematics education requires equity – high expectations and strong support for all students

[4] In science an attempt was actually made in the 1990s to develop a national curriculum framework (CMEC, 1997)

- Curriculum
    - A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades
- Teaching
    - Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.
- Learning
    - Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.
- Assessment
    - Assessment should support the learning of important mathematics and furnish useful information to both teachers and students.
- Technology
    - Technology is essential in teaching and learning mathematics; it influences the mathematics that enhances student learning.

Curriculum content is embodied in five strands, labeled somewhat differently from those found in the 1989 document. These are:

- Number and Operations
- Algebra
- Geometry
- Measurement
- Data Analysis and Probability

Some of these labels more closely resemble traditional branches of school mathematics and may be the reason some critics have come to believe that NCTM has repudiated its earlier emphasis on problem-solving as the core component and has embraced a back to basics approach to curriculum. However, the process standards originally identified in the 1989 document were reiterated in 2000 in more explicit form as follows:

- Problem solving
- Reasoning and proof
- Communication
- Connections
- Representation

These processes highlight the emphasis on ways of acquiring knowledge and require that the process-oriented approaches to teaching so widely decried by the critics be maintained. Our own reading of the 2000 document is that it represents more of a clarification and streamlining of *the Standards* than a fundamental shift in thinking.

An interesting addition to the document in 2000 is a chart indicating the relative emphasis on each of the content strands at the various stages of schooling. In the early grades, the strongest emphasis is obviously on number, with geometry also receiving high emphasis. Emphasis on algebra is minimal in the early grades and progressively increases through the grades to be the strongest strand at the high school level. Geometry retains fairly constant emphasis throughout. The emphasis on measurement and data analysis and probability are smaller throughout, with measurement dropping off in the middle years to the smallest component.

This has significant implications for the local curriculum. Following the 2000 *Principles and Standards* would reduce the demand to treat all strands fairly equally and would lead to increased emphasis on numeracy in the early grades. This should result in a reduced total number of outcomes and a stronger emphasis in areas which are perceived by teachers and other stakeholders as treated in insufficient depth in the current program. This pattern is evident in the revisions recently completed by the WNCP.

The *Principles and Standards* identify specific expectations for Grades Pre-K-2, 3-5, 6-8 and 9-12. One of the features of this division for local purposes is that it does not correspond directly to the divisional organization of our schools. This makes it more difficult to link key stage outcomes for local students with the NCTM divisions. The *Principles and Standards* document is complex, with its combination of content and process strands. Some of the specific content standards are difficult to interpret in terms of the depth of treatment required. For example a statement such as "select and use appropriate statistical methods to analyze data" (data analysis and probability, Grades Pre-K-2) gives little indication of the kinds of statistical methods appropriate for students at this level. This would obviously be different from what would be appropriate for higher grades, where the same statement again appears. Such statements are given meaning only through specific examples.

The content and process standards represent a two-dimensional structure, in which the process standards presumably are taught through the content. It might be expected that this would be depicted in a "table of specifications" embedding both content and process in examples or exercises. Instead, the content and process standards are essentially kept separate, encouraging an approach in which the processes can be treated as almost content-independent.[5]

In response to concerns that the *Standards* were not providing clear enough guidance for curriculum development, especially in terms of scope and sequence and in identifying the highest priority areas, the NCTM in 2006 produced a much smaller document called *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics* (NCTM, 2006). This document primarily targets content, more explicitly identifies the content areas that are of greatest importance at specific grade levels and presents a more targeted grade-based sequence for content learning.

While the *Curriculum Focal Points* emphasize that the process standards remain essential and that mathematics should be taught through a search for meaning, content is placed front and centre. Furthermore, it is made much clearer than in the 2000 *Principles and Standards* document that developing fluency in basic skills is a core requirement in the early grades. This is not missing in the *Principles and Standards* but is not highlighted as core. For example, a Pre-K-2 Standard for number and operations is *compute fluently and make reasonable estimates*. The focal points for grade 2 are; *develop an understanding of the base-ten numeration system and place-value concepts* and *develop quick recall of addition facts and related subtraction facts and fluency with multidigit addition and subtraction*. The latter leaves little doubt that fluency in addition and subtraction is the crucial outcome.

[5] It is interesting to note that a similar problem was encountered in science curriculum development in the 1960s, when the emphasis on science as a process was, in some programs, reflected in exercises which had no content of scientific interest but which was designed simply to illustrate the process.

In our view, the 2006 *Curriculum Focal Points* provide a basis for rebalancing of content emphasis in mathematics in a way which avoids the criticism of neglect of basic skills. While it may be argued that neglect of basic skills was never intended, it is certainly possible to read the two *Standards* documents as including basic skills among many other outcomes, with no particular order of importance. Indeed, the 1989 document explicitly identified this area as receiving reduced emphasis. The risk is that this might engender yet another "back to basics" pendulum swing. However, interpreted appropriately by curriculum developers as providing guidance to what is most important, and used in conjunction with the *Principles and Standards*, the *Focal Points* could potentially help achieve the balance that most teachers seem to be seeking and reduce the criticism of "too many outcomes" with no indication of any order of priority. What is important is that the *Focal Points* can serve to streamline the curriculum and provide scope and sequence. Curriculum guides, textbooks and other resources will continue to carry the burden of elaborating the program and to provide appropriate learning activities reflecting the *Standards*.

The WNCP Research Study

A major study of mathematics was commissioned by the WNCP in 2003 and completed in 2004 (Hold Fast Consultants, 2004). This study reviewed the literature in mathematics education and conducted surveys of the major stakeholder groups that are part of WNCP. Some of the previous material in this chapter was drawn from that document. However, that study requires more direct attention because it has become the starting point for the revisions to the common curriculum framework for Western and Northern Canada

This study pointed to the following major issues.

- Too much content for allotted instructional time
- Significant research has been done regarding the teaching and learning of mathematics since 1996
- Post-secondary acceptance of secondary courses
- Transitions between grades

The study recommended the following changes to the mathematics framework:

- Teach fewer topics in more depth

- Group outcomes that address similar concepts
- Avoid outcomes that are not mathematical or are addressed in other subjects
- Clarify outcome wording and provide a means allowing for better interpretation of the outcomes
- Increase focus on early numeracy
- Introduce pre-algebra earlier
- Introduce some topics later
- Ensure the flow of concept development
- Use terminology consistently

More specifically, with respect to the early years, the study recommended that the curriculum for Kindergarten to Grade 3 be focused on Number and Measurement only (this is a relatively extreme interpretation of the NCTM emphasis chart); and that the K–3 curriculum be based upon conceptual understanding with algorithms introduced only when conceptual understanding supports them.

It is interesting to note that there is no direct reference to the NCTM *Curriculum Focal Points* in the WNCP report. Indeed, this review predated the publication of the *Focal Points*. However, the move towards reducing the number of outcomes, clarifying wording and increasing the focus on early numeracy are entirely consistent with the *Focal Points* and seem to have anticipated the thrust of this document.

With respect to the high school program, the study reiterated the need for three distinct programs at that level. These programs were identified with the following types of students:

- Those entering post-secondary programs that require calculus (e.g., Mathematics, Sciences, Engineering, Commerce, etc.);
- Those entering post-secondary programs that do not require calculus (e.g.,Humanities, Fine Arts, some Trades and Technical programs, etc.); and,
- Those entering the workforce, trades or technical programs that do not require advanced mathematics.

It is important to note that the first of these programs is directed at a relatively narrowly-defined group, namely those planning to take post-secondary programs requiring calculus. Although the advanced mathematics program in the local curriculum is not so explicitly targeted, in practice students taking advance mathematics are the one found in the first-year university calculus courses The third level is identified more explicitly as directed to a specific type of student. This type of targeting is not found in the local program, where the general mathematics courses are designed to achieve many of the same outcomes as the other two programs, but at a lower level. This is designed to facilitate movement to the higher level courses, something which is not part of the WNCP report recommendations (and which rarely occurs in the local context in any case).

Factors Influencing Teaching and Learning in Mathematics

The remainder of this chapter is devoted to a review of what research tells us (and does not tell us) about the factors which influence teaching and learning. The focus here is quite pragmatic, dealing with factors that we might be able to do something about within the education system. We therefore do not emphasize socio-economic or cultural forces influencing outcomes. While the focus here is on mathematics, much of what we know is generic and applicable to other areas of the curriculum.

First, it is important that expectations for this part of the review not be set too high. Despite several decades of research on teaching and learning, robust results which would allow us to describe, much less prescribe, ways of teaching that can improve learning are hard to come by. Recent large scale surveys, such as PISA 2000 and 2003 (OECD, 2001, 2004) and SAIP Mathematics III (CMEC, 2001) have shown that many factors exert a measurable but small effect on learning and that no single factor is decisive. These surveys tend to show that socioeconomic factors are among the strongest influences on learning. However, most such studies do not investigate teaching and learning strategies over a student's entire school career and hence likely underestimate the influence of the latter on achievement.

It is difficult to find the right constellation of factors in any system, school or classroom, applied consistently enough across a student's school career to make a large difference and extraordinarily difficult to design studies which would isolate the specific factors likely to have the greatest impact. While this may sound discouraging, the goal of policy ought to be to bring about incremental shifts in the directions known to have positive effects and to attempt to align the desirable factors to bring about a cumulative effect on learning.

Despite the above comments, there is evidence that consistent exposure to the best teaching and learning strategies can make a large difference. This is shown in major syntheses by Wang, Haertel and Walberg (1993), Marzano (2003) and Scheerens and Bosker (1997). The problem is ensuring that these strategies are applied at the classroom level and are sustained over a student's school career.

In their well-known synthesis of factors influencing achievement, Wang, Haertel and Walberg (1993) advanced the concept of proximity as a way of thinking about the effects of various factors. The general hypothesis is that proximal factors, or those which touch most closely on the day-to-day lives of students, are likely to be more influential than more distal factors, such as district and state policies.

According to Wang, Haertel and Walberg, classroom management, meta-cognitive processes, cognitive processes, home environment, parental support and student/teacher social interactions showed stronger relationships to achievement than broad state and district level educational policies. This point is of crucial importance because it suggests that broad policy initiatives are likely to result in improved learning only if they can be translated into change at the individual teacher or student level.

Scheerens and Bosker (1997) produced a ranking of school factors found to have positive influences on learning. These include time, monitoring, pressure to achieve, parental involvement and content coverage. The type of school climate most likely to enhance learning is an orderly atmosphere, rules and regulations and good student conduct and behaviour. Similarly, effective classroom management strategies include direct instruction, monitoring student progress and positive work attitude. In an independent review, Marzano (2003) independently developed a list which is almost identical to that of Scheerens and Bosker.

During the 1960s and 1970s, a considerable amount of research was conducted on student-teacher interactions and other aspects of classroom teaching and learning. This research has been synthesized by such authors as Brophy and Good (1986) and Brophy (1997). The key enabling condition is defined as "opportunity to learn." While it seems common sense, even obvious, to state that greater opportunity to learn should lead to greater learning, the significant issue for both research and practice is how to maximize opportunity to learn.

In general terms, the factors most likely to maximize opportunity to learn may be summarized as follows:

- Having adequate time available for learning
- Maximizing teacher and student engagement during this time
- Ensuring a positive disciplinary climate which avoids disruption and loss of time
- Engaging in academically meaningful work at a moderate level of difficulty
- Having high expectations
- Maximizing content coverage
- Monitoring learning and using assessment to enhance learning

The well known time-based model proposed by Carroll (1963) captures the notion that teaching influences learning by incorporating the components of opportunity to learn, time allocated by the teacher and quality of instruction into the core model. This model is further linked to a broad approach to teaching and to educational policy in its extension by Bloom (1976) to the concept of "mastery learning." In an attempt to address the issue of equity in learning, Bloom proposes that time be varied sufficiently to allow almost all students to achieve specified learning outcomes. This, of course, requires significant variation in both school organisation and teaching strategies. It is difficult to find examples of large-scale implementation of mastery learning, despite its strong research support.

Because the school day and year are fixed, one of the most common ways of expanding the time allocated to learning is homework. While homework is an established feature of schooling and seems to have become more prevalent with increased emphasis on outcomes, it is not without controversy, as indicated by the comments made in the focus groups and submissions to this study.

A recent comprehensive review of the effects of homework on academic achievement is available (Cooper, Robinson & Patall, 2006). Of more than 900 empirical studies conducted between 1987 and 2003, about 75 met the selection criteria established by the reviewers. These studies were reviewed using established qualitative and quantitative synthesis methods. Most studies referred to homework in either language arts or mathematics.

The results showed the effects of homework to be generally positive. Effects are very small at the elementary level but increase at higher grades. Like other factors affecting learning, the effects are not large enough to make a decisive difference for most students but can certainly contribute to a difference between pass and fail for a marginal student. Limited information was available from the research on the optimum amount of homework but this suggests that the upper range for high school students is between 90 and 150 minutes.

Data on homework from the most recent School Achievement Indicators Program (SAIP) mathematics assessment (CMEC, 2003), as reported in the next section, are consistent with the results given in the Cooper, et. al. review.

The most comprehensive recent review of factors affecting mathematics learning seems to be that by Hiebert and Grouws (2007). These authors examine the evidence available to support the claim that "the nature of mathematics teaching significantly affects the nature of students' learning." After reviewing the early research and concluding that opportunity to learn is the key requirement, Hiebert and Grouws focus more specifically on the distinction made earlier between teaching for meaning and teaching for skill development. Hiebert and Grouws argue that much of the research on teaching has been more concerned with skill development than with conceptual understanding and that the teaching techniques most effective for the former are not necessarily the most appropriate for the latter.

Citing the well known Trends in International Mathematics and Science (TIMSS) video study (Stigler & Hiebert, 1999), Hiebert and Grouws point out that mathematics teaching in some of the highest achieving countries is characterized by classroom interactions designed to focus on conceptual meaning. In contrast, mathematics classes in the United States and Australia are characterized by attention to lower level skill development.[6] Many other studies are cited which suggest that teaching for conceptual development is associated with higher achievement and that such teaching can also enhance skill development.

Hiebert and Grouws give no explicit prescriptions for classroom teaching practices. However, they argue that we can apply the admittedly incomplete knowledge base in some useful ways immediately while other areas require further research. The following are among the points which seem to have immediate application:

- Teaching for conceptual meaning and teaching for skill development are not contradictory. Teaching which focuses on meaning is likely also to enhance skill development.
- There is a need to focus on teaching, not on teachers. General characteristics of teachers and teacher qualifications are not associated with student outcomes. The focus needs to be on what teachers and students do.
- It is important to be explicit about learning goals. (We note that this is one area in which there has been significant improvement in recent years).

The SAIP and PISA Mathematics Studies

These studies warrant some specific comment because they are among the few for which specific Canadian results are available and because these are large scale high quality comparative studies capable of yielding results that are reasonably generalizable to Canadian and provincial populations of students. The focus here is on the factors influencing mathematics performance. Actual achievement results for this province compared to other jurisdictions are presented in a later chapter.

[6] No Canadian schools were included in the TIMSS video study. It seems reasonable to surmise that Canadian schools are more like those in the United States or Australia than like those in the Netherlands, Japan or Hong Kong. Indeed, the PISA study indicates that schools in English-speaking countries are more similar to each other than they are to schools in other countries. Nevertheless, it is important to point out that in recent international assessments Canadian students do better than those in either the United States or Australia.

The 2003 Programme for International Student Assessment (PISA) (Bussière, Cartwright & Knighton, 2004)) assessed the mathematics performance of 15-year-old students in more than 40 countries, mostly the developed countries who are members of the Organization for Economic Cooperation and Development (OECD). In Canada, the sample sizes used allowed provincial comparisons of achievement and of the relationship between achievement and various teaching and learning strategy indices. These indices involved measures of what was called “student engagement in mathematics.” Some of the major results for Canada, with comments on Newfoundland and Labrador, may be summarized as follows:

- Students using high levels of all of the three indicators of engagement in mathematics learning, labeled memorization, elaboration and control, had higher levels of achievement than those with low levels of engagement. Although these indicators may seem contradictory (e.g. memorization may be associated with basic skills and elaboration with conceptual understanding), all showed positive effects.

- Students in Newfoundland and Labrador showed high levels of use of memorization and control strategies but were in the mid-range among provinces in their use of elaboration strategies.

- Preference for cooperative learning situations showed a negative association and preference for competitive learning situations a positive association with achievement.

- Relative to other provinces, students in Newfoundland and Labrador showed the highest preference for cooperative learning. However, students in this province were in the middle of the range in preference for competitive learning. The highest performing provinces, Alberta and Quebec were also highest on competitive learning.

- Positive attitudes towards mathematics, including interest in mathematics, belief in its usefulness, perceived ability and mathematics confidence were all positively associated while mathematics anxiety was negatively associated with achievement.

- Students in Newfoundland and Labrador shared with those in the higher achieving provinces relatively high positive scores on most of the positive attitude indices and a relatively low mathematics anxiety score.

- Despite these positive indicators, the average achievement of students in this province was in the lower range of the distribution among provinces. (Comparative details will be presented later). This seems to reflect the high impact of socioeconomic indicators on achievement and the relatively low levels of students in this province on socioeconomic indicators.

The School Achievement Indicators Program (SAIP) is a pan-Canadian assessment of students at ages 13 and 16. The most recent of three SAIP mathematics assessments, conducted in 2001, also gave some detailed results linking mathematics achievement to student background and aspirations and to teaching and learning strategies. Because these are more complex than the PISA results, a summary is given in Table 2.1.

It is important to note that all of these are correlational results, which do not necessarily point directly to causes of higher or lower achievement. The class size result is an obvious example of this. It is likely that this result is a consequence of larger classes being concentrated in urban areas, where other factors contribute to higher achievement. The class size effect is thus confounded with these other factors. On the other hand, although more highly controlled studies of class size indicate that smaller classes yield improved achievement in the primary grades, it is clear from the SAIP and other studies that class size is not an overriding influence on achievement, whatever the value of smaller classes for other purposes.

Table 2.1
Factors Associated with Mathematics Achievement:
SAIP Mathematics Assessment, 2001

Positive Effects	Negative Effects
Student effects • Mother's education • Planning to attend university and to work in a field requiring mathematics • Time on mathematics homework • Persistence in solving mathematics problems • Teacher gives notes • Teacher shows how to do problems • Teacher assigns homework • Work on textbook exercises • Students ask questions • Students use calculators	**Student effects** • Taking mathematics tutoring • Perceived difficulty of mathematics • Attribution of poor marks to bad luck • Days absent from school • Work with parents on homework • Doing mathematics projects • Working in small groups • Off-topic discussion • Losing 5-10 minutes because of disruptions • Using books and magazines other than textbooks • Using computers • Using slides, videos, films
School effects • Larger schools and larger communities • Larger class sizes	**School effects** • Limitations on instruction due to student backgrounds, diversity, resources, community conditions

The negative SAIP results for tutoring are interesting in light of the seemingly widespread use of tutoring in mathematics. However, like class size, the tutoring effect is likely confounded with other factors, such as the performance of students prior to their using a tutor. The most likely explanation of this result is that students who are tutored are likely to be performing at low levels, otherwise they would not need a tutor. While tutoring may have some positive effects on school grades, it is unlikely that it transforms low achieving students into high achieving ones or that it has any impact on the type of broader mathematical literacy measured by large scale assessments. We were not able to locate a recent synthesis of research on mathematics tutoring, so its overall impact is not known. However, it is interesting to note that tutoring, like homework, is consistent with the idea of allocating additional time to students who need it. The effects of tutoring, controlling for other factors, might be expected to be at least as great as those for homework.

Teacher Professional Development

As the WNCP research study states, “what teachers and students are able to do together in mathematics classes is at the heart of mathematics education.” (Hold Fast Consultants, 2004, p.51). These authors argue that teacher knowledge and beliefs about mathematics is key to their behaviour and that differences in the performance of, for example, Asian compared to American students is related to differences in teachers fundamental understanding of the nature of mathematics and mathematics learning. Finally these authors make the point that curriculum change is insufficient unless also accompanies by changes in teacher beliefs and behaviours. It is clear that the more constructivist approach precipitated by both the 1989 *Standards* and the 2000 *Principles and Standards* required a significant shift in thinking about the nature of mathematics teaching and learning. The rebalancing of priorities embodied in programs based on the *Focal Points* calls for yet further rethinking of what we are about in mathematics teaching.

All of this raises the issue of what extent and type of teacher professional development is required to ensure that teachers can cope adequately with new programs and with continuously evolving thinking about curriculum.

An extensive literature on teacher professional development exists. The most comprehensive synthesis we could find (Dall'Alba and Sandberg, 2006) focused on models for career-long development of teacher expertise and says nothing about specific programs. At the other extreme, there are many small scale studies in mathematics, mainly focusing on short-term intervention at specific grade levels (e.g. Ross, Hogaboam-Gray & Bruce, 2006) or on specific topics (Jacobs, Franke, Carpenter, Levi & Battey, 2007). A recent thrust in this research has been in the use of information and communications technologies as vehicles for mathematics professional development (e.g. Miller & Glover, 2007). This is part of a broader trend towards developing what has become known as "professional learning communities." This idea encompasses such practices as induction programs for new teachers, teamwork within schools and between schools and teacher education institutions and the use of distance learning technologies for professional development. The Virtual Teacher Centre recently established in this province is an example of an initiative which uses technology to develop and promote teacher development.

The WNCP research study also commented on the issue of teachers' mathematical knowledge. Citing work by Ma (1999) and others, the authors make a case that the problem of changing practices is compounded when teachers lack a profound understanding of mathematics. They also make the point that it is necessary to help teachers build their own understanding of the underlying mathematical concepts and pedagogical approaches if we wish to ensure that such understanding is developed in children. In the end, the WNCP study argues that professional development must be a continuous process and an integral component of curriculum change.

Unfortunately, we were not able to locate a synthesis of the professional development literature which points to specific professional development approaches that are likely to have the most impact. Many of the more comprehensive models would required a significant restructuring of how schools and teachers function and a different relationship between teachers, teacher educators and other professionals concerned with improvement.

There is little research support for any particular approach to the more immediate problem, that of developing a professional development model in support of the further changes to the mathematics curriculum to be outlined in this report. Nevertheless, several principles seem to follow from the above points:

- Professional development must be an integral part of the curriculum implementation process.
- Professional development needs to move from the one-shot workshop approach to a more continuous process.
- Professional development needs to focus on those teachers with the lowest levels of mathematical knowledge and experience in teaching mathematics.

In practice, it may not matter much whether professional development is conducted in the form of summer institutes, school closeouts or pulling teachers from classrooms through the use of substitute teachers. The challenge is to find ways of reaching all teachers who are expected to be involved in curriculum change, using methods tailored to teacher prior background and experience achieving greater continuity and developing a means of follow-up to any initial experience. Fortunately, the appointment of a new corps of mathematics (numeracy) support teachers along with other components of the Department of Education's *Excellence in Mathematics Strategy* gives some scope to develop and implement a more comprehensive approach to mathematics teacher professional development than has previously been available. Some proposals for such a strategy are presented later in this report.

It is worth noting that a 2001-02 survey of teachers in this province revealed that curriculum issues were identified by far more teachers than any other as an area needing professional development. Within curriculum, mathematics was identified by more teachers than any other subject (Department of Education, 2004).

What Does the Research Tell Us About Mathematics Curriculum, Teaching and Learning?

One conclusion that may be drawn from the literature is that mathematics education suffers from a surfeit of sometimes conflicting theories of learning but lacks a coherent theory of teaching. Behaviourist and Gestalt theories and their successors contribute to but do not resolve the controversy over teaching basic skills versus teaching for meaning. On the other hand, the large number of teaching strategy and engagement factors which seem to influence learning in small ways have not been brought together into a coherent model which can provide teachers with good advice on how they should function day to day in the classroom.

While there is clear evidence that running an orderly classroom which minimizes disruption and maximizes on-task behaviour contributes to higher achievement, there is unease among some educators that this results in excessive uniformity and a narrowing of goals and does not contribute to what may be called more meaningful learning. We are not in a position to prescribe in any detail a set of teaching strategies which will maximize the probability of students achieving at the highest levels, especially in developing mathematical meaning.

Added to this, and especially applicable to this province, is the evidence that factors outside the school play a significant role in achievement. It seems likely that the positive factors among students in this province evident from the SAIP and PISA studies are being offset by home and family backgrounds and perhaps further exacerbated by out-migration, enrolment decline in rural schools and other strong social forces operating within the province.

Despite the curriculum controversies, our sense is that the prospects for curriculum improvement are considerably better now than they were a few years ago. The clarification offered by the NCTM *Focal Points* goes a long way towards bridging the gap between the advocates of basic skills and those who believe that teaching for meaning is more important than teaching for basic skill development. There may be relatively few at either extreme of this continuum and it is unlikely that any curriculum would meet the approval of those at the extremes. The message from the literature and, as we shall see, from the comparative analysis and the consultations, is for simplifying and rebalancing the curriculum.

The message from the professional development literature is less encouraging. There is little to indicate that small scale initiatives are of much value. Indeed, if we had to generalize from most small scale studies in mathematics, we would have to argue for multiple workshops focusing on very small curriculum components. At the other extreme, the broader literature argues for almost a career-long process for the development of expertise. For the current study, the professional development issue must be narrowed down to what is required to support mathematics curriculum change and what is feasible to implement with the resources likely to be available and without undue disruption of school activities or excessive loss of instructional time.

III COMPARATIVE CURRICULUM ANALYSIS

The comparative curriculum analysis was the largest single component of this study. Detailed comparisons, at the level of Specific Curriculum Outcomes, were made directly for Grades K-9 for Newfoundland and Labrador, Ontario and the Western provinces and territories. In the latter case, two versions were available, based on the Western Canadian Protocol of 1995 and the Western and Northern Canadian Protocol published in 2006. In the case of senior high school programs, it was not possible to make such detailed comparisons because high school courses are differentiated at three levels and the outcomes addressed in courses across the jurisdictions vary significantly across courses. Instead, we compared "topics" within individual courses. In some cases, these corresponded to course specialization by strands. In others, the topics were more typically expressed in more conventional form such as "quadratics" or "functions."

In addition to the interprovincial comparisons, we examined outcome statements found for England, Wales and New South Wales in Australia.[7] Outcomes for other Australian states were available but were judged similar to those for New South Wales. In this case, the detailed comparisons were limited to Grade 6, with a more cursory examination of other grades. The results suggest that these other countries are taking much the same approach to mathematics as is found in Canada. Students in all of these countries are reasonably high performing on international assessments. None are as high as Canada as a whole but most are within the same general range as students in this province.

[7] The original intention was to look at programs in a few high performing countries such as Finland, Japan and Korea. However, we were not able to find detailed curriculum documents in English for these countries.

K-9 Comparisons

The interprovincial comparisons for Grades K-9 were completed initially by members of the research team. These were then verified by an expert panel of six numeracy support teachers (two at each level) from around the province. Panel members were asked to review the work of the original analysts rather than conducting independent separate analyses. In addition to matching the outcomes, panel members were asked to make a broader judgment on the breadth and depth of curriculum in other jurisdictions compared to the local program.

The detailed K-9 comparisons are quite lengthy and are reported in the Appendix. In this section an attempt is made to summarize the main results of the comparisons and to offer a few comments on these results.

Table 3.1 gives the total number of outcome statements for each of the programs reviewed. While this gives only a limited picture of the scope of the curriculum, it is relevant because it speaks to one of the major criticisms we heard about the curriculum in this province, namely that there are too many outcomes.

Table 3.1
Number of Outcomes by Program and Grade

Grade	NL	Ontario	WCP 95	WCNP 06
K	46	28	24	9
1	44	56	41	18
2	59	64	53	25
3	50	65	61	25
4	64	74	54	24
5	64	63	56	25
6	65	59	54	26
7	70	60	55	25
8	58	61	45	19
9	54	52	51	22
Total	574	582	494	218

The table shows that the total number of outcomes for K-9 is about the same as in Ontario, is slightly more than in the current Western programs represented by WCP 1995 and is substantially more than in the WNCP 2006, which forms the basis for current curriculum development work in the Western and Northern jurisdictions. Interestingly, the largest single difference is in Kindergarten, where this province has double or more the number of outcomes found in other jurisdictions. It is not clear why this is so. The most direct example of outcomes in the local Kindergarten program that are not found elsewhere are those having to do with time (e.g. morning, afternoon, night; past, present and future; days, weeks, months). These are examples of outcomes which might appropriately be treated in areas of the curriculum other than mathematics.

Table 3.2 is more complex but allows a rough estimate of the “degree of match” between the various programs. Compared to Ontario, in both directions, it appears that this province has 75% or more matching outcomes. Relative to the WCP 1995, there are more outcomes in the local program that are not in WCP than the other way around (i.e. outcomes in WCP 1995 that are not in NL). This is not a particularly bad match considering that the programs were developed independently. There is little to indicate that the program in this province (or the Atlantic provinces generally) is dramatically different from those currently in place elsewhere in Canada. While WNCP 2006 seems to provide a basis for a fairly significant shift in Western and Northern jurisdictions, this is not yet in place.

One thing that may account for some of the mismatch is that some of the local outcome statements are worded in such generic terms that it is difficult to determine what is being said. Examples are “discuss and interpret displayed data” (K-C2) or “continue to solve a wide variety of measurement problems” (3-D8) and “select and use appropriate strategies in problem situations” (9-B15). There are not a lot of such statements but in none of these cases was it possible to find a match because the statements are not sufficiently clear.

The verification exercise by the expert panel yielded less than 10% disagreement overall with the original matching. In most cases, some specific outcomes were judged to be classified inappropriately by one but not both reviewers. In light of this, there seemed to be little point in changing the original classification. While this might have yielded changes of one or two outcomes in specific cells of Table 3.2, these would not have changed the overall pattern given.

Table 3.2
Degree of Match of Outcomes by Program and Grade

Grade	NL not in Ontario	NL not in WCP 95	NL not in WCNP 2006	Ontario not in NL	WCP 95 not in NL	WCNP 06 not in NL
K	7	18	28	3	2	0
1	1	6	16	6	10	1
2	7	7	22	11	9	1
3	11	13	21	19	25	2
4	17	21	35	14	13	5
5	12	21	34	12	13	1
6	23	18	25	5	10	3
7	33	19	47	29	13	7
8	16	14	23	18	7	0
9	30	22	33	16	12	4
Total	124	140	237	104	101	17

These tables show that the key difference is between WNCP 2006 and all of the other programs. In particular, a large number of outcomes found in the local program are not present in WNCP 2006. This leads to the question of what changes were made in developing that framework and, in particular, whether this represents a significant reduction in content or simply a consolidation of outcome statements. This question was answered in the summary given in the previous chapter. There has been a clear focus on reducing the number of topics and on treating the remaining topics in greater depth. In the early years, the focus has shifted significantly to the number strand with number concepts and number operations being combined. The algebra strand has been strengthened with pre-algebra concepts being introduced in earlier grades. On the other hand, aspects of geometry and data analysis have been moved to later grades.

This is not the place to go into great detail on the specific changes in outcomes. The crucial point is that WNCP 2006 appears to address most of the concerns that have been expressed locally, in a manner which is consistent with the NCTM *Focal Points*. A solution to the problem of too many outcomes and lack of depth thus seems to be available. How to get to this solution is a point to be addressed in the final chapter.

High School Programs

The course structures at the senior high school level (Grades 10-12 or Levels I-III) are fairly complex and individual courses are not directly aligned with outcome statements in the same way as for the earlier grades. Also, in some other jurisdictions, the high school courses are more highly specialized and not directly comparable to the courses in this province. It was therefore necessary to take a different approach to comparison. Rather than attempting a direct outcome match, we first looked at the overall program structure in other jurisdictions. This was followed by a more detailed review of "topics" covered in the various courses offered at levels comparable to the local courses. While the topics approach is not as detailed as the outcomes analysis, this conveys a reasonable picture of what students in other jurisdictions are expected to know or be able to do at each grade level in the high school.

Table 3.3 gives the comparison of course structures. Again the distinction between the current WCP, represented here by the British Columbia program and the WNCP 2007, which outlines proposed courses for implementation in the WNCP jurisdictions beginning in 2010, should be noted.

All jurisdictions have some form of a three-level structure, similar to our Advanced, Academic and General programs. However, in other jurisdictions, the lowest level is more explicitly identified as practical or workplace mathematics. The important difference here is that our general mathematics course is based on essentially the same outcomes as the other courses, differing in depth of treatment. The practical mathematics in other jurisdictions is distinctly different from the academic streams. Some of our informants have indicated that the local structure was intended to allow students to move across the levels, and especially upward. However, this seems to be a rare occurrence.

Table 3.3
Comparison of High School Course Structures

Program Level	Course 1	Course 2	Course 3	Course 4	Course 5
Newfoundland and Labrador					
Advanced	Mathematics 1204	Mathematics 2205	Mathematics 3205	Mathematics 3207	Mathematics 4225
Academic		Mathematics 2204	Mathematics 3204	Mathematics 3103	
General	Mathematics 1206	Mathematics 2206	Mathematics 3206		
Note: The Newfoundland and Labrador courses are not strictly sequenced. For example, students may take the 2000 and 3000 level courses in either order.					
British Columbia					
Calculus Post-Secondary	Principles of Mathematics 10	Principles of Mathematics 11	Principles of Mathematics 12	Calculus (Challenge for Credit)	
Non-Calculus Post-Secondary	Applications of Mathematics 10	Applications of Mathematics 11	Applications of Mathematics 12		
Life, Business, Industry, and Government	Essentials of Mathematics 10	Essentials of Mathematics 11	Essentials of Mathematics 12		
WNCP 2007 (BC, AB, SK, MB, YK, NW, NT)					
Implementation	2010	2011	2012		
Precalculus	Foundations of Mathematics and Precalculus 10	Precalculus 11	Precalculus 12	Calculus (Challenge for Credit)	
Foundations of Mathematics		Foundations of Mathematics 11	Foundations of Mathematics 12		
Apprenticeship and Workplace	Apprenticeship and Workplace Mathematics 10	Apprenticeship and Workplace Mathematics 11	Apprenticeship and Workplace Mathematics 12		
Ontario					
University Preparation Courses	Principles of Mathematics 10	Functions 11	Advanced Functions 12	Calculus and Vectors 12	
			Math. of Data Management 12		
University/College Preparation Courses	Foundations of Mathematics 10	Functions and Applications 11	Mathematics for College Technology 12		
College Preparation Courses		Foundations for College Mathematics 11	Foundations for College Mathematics 12		
Workplace Preparation Courses		Mathematics for Work and Everyday Life 11	Mathematics for Work and Everyday Life 12		

It is also important to point out that the equivalent courses to our Advanced level are more explicitly labeled as designed to prepare students for calculus courses at the university level. This is a crucial difference which speaks to an issue of whether the academic program prepares students for such courses. In practice, we understand that few students now take the first semester calculus course at university without having done the Advanced program.

More generally, the labels given to the courses in other jurisdictions are more descriptive of the content and emphasis. Courses labeled "university preparation," "pre-calculus " or "apprenticeship/workplace" convey more meaning than the more generic terms used locally.

The current structure for British Columbia includes three courses at each of the three levels. The top two courses follow the WNCP structure while the "Life, Business, Industry, and Government" course is local to BC. The proposed new structure for the WNCP jurisdictions has two levels at Grade 10, as is now the case in this province, with the division being between the Apprenticeship and Workplace course and the combined Academic and Advanced equivalents.

No other jurisdictions have courses equivalent to our 3103 or 3207. Other jurisdictions do have fourth-level Calculus courses which appear more comparable to Mathematics 4225. There has been some criticism of 3103 on the grounds that the need for this course simply points to weakness in the courses at earlier levels. It can also be argued that the need for Math 3207 stems from the attempt to cover too much in Math 1204, which weakens the depth of treatment in that course. We do not have detailed information on the value of Math 3207 for university preparation. However, an argument can be made that having an Advanced course at Level 1 would allow students the depth needed from the beginning to avoid an additional Level III course.

The Ontario program is more complex than any of the others and may be seen as more strongly streamed. Ontario is the only jurisdiction with distinct university and college preparatory courses. Ontario also has two university preparation routes at Grade 12, separating advanced functions from data management. This is an interesting division as it provides a route to university entrance other than calculus preparation, one which seems attractive to the large number of students whose exposure to mathematics beyond school is much more likely to involve statistics and data management than calculus. There is a complex web of potential paths through these courses, but generally the pattern is that students can move downward but not upward.

Textbooks and Other Resources

Table 3.4 indicates the textbook series, publishers, date of publications, and the additional student and teacher resources of various K-12 Canadian mathematics curricula.

Intermediate. Grades 7 and 8 texts and resources in this province are not completely matched to the provincial curriculum guide. There are numerous curriculum outcomes for Grades 7 and 8 that are not addressed in the prescribed texts. Therefore, teachers must find alternate resources for supplementation of these outcomes by either using other books or by creating and sharing resources between teachers. For example, students in Grade 7 are expected to solve and create problems involving common factors, greatest common factors, common multiple and least common multiples. However, these outcomes are not addressed in the prescribed text.

High School. The local high school textbook series, consisting of Mathematical Modeling Books 1–4 and Constructing Mathematics Books 1–3, have a high emphasis on problem solving and low emphasis on skill development and practice. The Department of Education has created supplementary support materials for these texts to assist teachers across the province. The accompanying teacher resources have deficiencies by not providing separate solutions manuals, unit tests, extra practice black-line masters and assessment banks.

Atlantic Provinces. Nova Scotia is developing an intermediate series with McGraw Hill Ryerson called Mathematics Focus on Understanding. Grade 9 was published in 2006, and Grade 7 in 2007. There is no Grade 8 resource to date. Prince Edward Island uses the Western edition of the Mathpower series from McGraw-Hill Ryerson for Grades 7–12 and Pearson's Quest 2000 for K–6.

Teacher Resources. Table 3.4 indicates that local teacher resources available for Grades K-3 and 7-12 are more limited than those associated with the newer textbook series from various publishers across Canada. For example, the 2007 Math Makes Sense WNCP Grade 7 extra resources include a student CD with extra practice, a practice/homework book and a CD with practice and test generator. In addition, the teacher resources include a CD of planning charts, unit tests, assessment and extra practice masters, plus a DVD of projectable student book pages, teacher tips, and modifiable black line masters.

The Math Makes Sense series also includes resources for combined grades. Presently they are for Grades 1/2, 4/5, and 7/8. Another publisher which concentrates on Grades 7-12 is McGraw-Hill Ryerson. They have started a new series for the WNCP common curriculum called MathLinks. Additional Grade 7 resources include an exercise and homework book, black-line masters CD, solutions CD, computerized assessment bank, website, Webcasts, and professional development.

Table 3.4
Comparison of Textbooks and other Resources

Textbook Series Comparison				
Grade	Series	Publisher	Year	Resources
Newfoundland and Labrador				
K-3	Quest2000	Pearson/Addison Wesley	1996	Teacher's Guide and Journal
				Teacher Support Package
				Extra Practice and Testing Masters
				Practice and Homework Book
				Problem of the Week
4-6	Math Makes Sense (Atlantic)	Pearson/Addison Wesley	2005-2006	Teacher Guide (Binder, CD)
7-8	Minds on Math	Pearson/Addison Wesley	1996-1997	Teacher's Resource
				*Test Package (not provided by Department)
9	Minds on Math (Atlantic)	Pearson/Addison Wesley	2003	Teacher's Resource
10-12	Mathematical Modeling	Thompson/Nelson	2000-2002	Teacher's Resource
10-12	Constructing Mathematics	Thompson/Nelson	2000-2002	Teacher's Resource
Nova Scotia				
7-9	Mathematics Focus on Understanding	McGraw-Hill Ryerson	2006-2007	Teacher's Resource
				Solutions Manual
WNCP				
K-9	Math Makes Sense (WNCP)	Pearson/Addison Wesley	2007-2009	Pro-Guide (Binder, CD, and DVD)
				Big Math Books (K-2)
				Little Books (K-2)
				Audio Package CD (K-2)
				e-Tools (K-9)
				Practice and Homework Book (3-9)
				Together Resource: Support for Combined Grades (1/2, 4/5, 7/8)
				Extra Practice and Test Generator (7-9)
7-9	MathLinks	McGraw-Hill Ryerson	2007-2009	Teacher's Resource

				Exercise and Homework Book
				Blackline Masters CD
				Solutions CD
				Computerized Assessment Bank
				Website
				Professional Development
K-9	Math Focus	Thompson/Nelson	2007-2009	Teacher's Resource (Binder, CD)
				Student Workbook
				Solutions Manual
				Computerized Assessment Bank
				Student Success Workbook
				Combined Grades Resource
10-12	Mathpower	McGraw-Hill Ryerson	1998-2000	Teacher's Resource
				Blackline Masters
				Power Slides CD
				Solutions
				Computer Data Bank
				Computerized Assessment Bank
10-12	Pure Mathematics	Pearson/Addison Wesley	1998-2000	Teacher's Resource
				Independent Study Guide
				Template and Data Kit CD
				Test Manager CD
10-12	Applied Mathematics	Pearson/Addison Wesley	1998-2000	Teacher's Resource, Project Book, Technology Kit
				Project Book
				Technology Kit
Ontario				
K-8	Math Makes Sense	Pearson/Addison Wesley	2004-2006	Teacher Guide (Binder, CD)
				Practice and Homework Book (3-8)
				Extra Practice and Test Generator (7/8)
				Together Resource: Support for Combined Grades (1/2, 4/5, 7/8)
				e-Tools (K-8)
				E-Text (7/8)
				Big Math Books (K-2)
				Little Books (K-2)
				Audio Package CD (K-2)

7-8	Making Connections	McGraw-Hill Ryerson	2004	Teacher's Resource (Binder, CD)
				Workbook
				Computerized Test Bank
				Solutions CD
				Website
K-8	Nelson Mathematics	Thompson/Nelson	2004-2006	Teacher's Resource (Binder, CD)
				Student Workbook
				Combined Grades Resource
				Computerized Test Bank (7-8)
				Solutions Manual (7-8)
				Website
				Math Activity Workbook (1-2)
				Game Cards (1-2)
				Audio CD (K-2)
				Little Books (K-2)
				Poster Pack (K-2)
K-1	MathWorks	Scholastic	2006	Teacher Guide

IV CONSULTATIONS

The consultative component of the study consisted of a series of focus group sessions, a call for submissions and a teacher survey. Results from these activities are reported in this chapter. The survey results are given here in summary form. A background report giving the detailed survey results has been prepared and is submitted separately.

Submissions

A total of 18 submissions were received from a variety of sources including parents teachers and principals, individual Memorial University faculty members, the Department of Mathematics and Statistics at Memorial University and provincial organizations including the Newfoundland and Labrador Federation of School Councils, The Newfoundland and Labrador Teachers Association and the Newfoundland and Labrador School Boards Federation.

It would be inappropriate to draw any strong inferences from such a small number of submissions. Nevertheless, some of the themes which emerged are consistent with the data from other sources. These may be summarized as follows:

- There are too many outcomes and not enough time.
- The program neglects the basics (typically the reference was to multiplication facts and the common algorithms for the basic arithmetic operations).
- It is difficult for students, especially those with language difficulties, to have to explain their strategies for everything.
- Textbooks are not matched to outcomes and are unappealing to young children.
- Parents as well as children are struggling with this approach to mathematics and parents are frustrated at not being able to help their children.

- There is too much homework in mathematics. What cannot be done in class is sent home.
- More emphasis is needed on professional development for teachers.
- There is a need to develop "automaticity" in basic operations before students can move to advanced operations or problem solving.

A few respondents also referred to the challenge the investigative approach poses for teachers. This comment was sometimes followed by a plea for more professional development but in other cases this was seen as resistance to change and a tendency to revert to old approaches.

Focus Groups

Focus group sessions were held in each of the four Anglophone school district offices[8.] Each group had 12-15 participants, mainly teachers from each level, numeracy support teachers, school district mathematics specialists and typically one or more senior officials of the district. Parents (School Councils) were also represented at some but not all of the sessions. In addition to the district sessions a focus group was held in St John's involving representatives of the Newfoundland and Labrador Teachers Association, the Newfoundland and Labrador School Boards Federation and the Newfoundland and Labrador Federation of School Councils. Finally, sessions were held with individuals from the Mathematics Department and the Faculty of Education at Memorial University and from the College of the North Atlantic and the Marine Institute.

The themes identified above were also evident in the focus group discussions. However, participants in these discussions were able to give much more elaborated accounts of the issues and to illustrate with specific examples from their experience. Some themes also emerged that were not raised in the submissions. Because bullet points cannot capture the richness of the discussion, a more narrative account is attempted, under several main headings.

8 The Conseil scolaire francophone was not considered part of this study because that board follows a different curriculum.

Too Many Outcomes. This was clearly the dominant theme in all of the discussions. More respondents referred to this than to any other issue. Some elaborated by arguing that the consequence is lack of depth, inability to internalize the concepts and the necessity for such a fast pace that there is no time for review or consolidation. Others referred to this as a cause of weakness in basic skills that students at specific levels should be expected to know. No matter what the level, concerns were expressed that students are advancing without meeting expectations.

Teachers are quite aware of the documentation on reducing the number of outcomes that went to schools at the beginning of this school year and most welcome these changes. Some appear to interpret this as the Department listening to teacher concerns. Others see this as a genuine reduction in expectations while others seem to see the changes more as a way of reducing duplication.

Match of Textbooks to Outcomes. This theme was closely related to the first, in that teachers concerned with the number of outcomes tended to follow this with comments to the effect that the problem is exacerbated by the lack of match of textbooks to the outcomes. This issue was interpreted in different ways by different respondents. Some felt that the textbook is only one of many resources and that we should not worry much if the match is less than perfect. Others were of the view that, since the text is the only resource available to everyone, including students and parents, this resource should align closely with outcomes.

Some of the other complaints about textbooks concerned their overall quality and appeal to students. Comments in this area had to do with errors in content, lack of practice exercises and lack of sufficient explanation to support a conceptual approach. Several respondents referred to the activities of a few local teachers in producing workbooks and teaching guides designed to bridge the textbook gap. There were complaints that students are being encouraged to purchase these resources, thus undermining the free textbook program. Respondents were not of one mind on the quality of these resources, with some seeing these as necessary evils, others as the answer to the textbook problem, and a few going as far as to recommend that the Department purchase these documents. Some teachers described efforts to photocopy large amounts of text material as another way of overcoming textbook limitations and complained that teachers should not have to do this kind of work.

The most pointed criticism of textbooks centered around the books for Grades 7 and 8. These were widely condemned by teachers involved with these grade levels and references to the need for supplementary resources were more common for these grades than for others. The Grade 9 book, on the other hand, was generally viewed more positively.

Other Resources. Generally positive comments were made about the mathematics curriculum guides. Other resources received little comment. There were a few concerns that manipulatives are not properly organized and not being properly used and that working with manipulative is time-consuming. Other comments about manipulatives were supportive and consistent with the sentiments of some respondents that this is an essential component of the current approach to mathematics.

Only a few comments were made about the use of calculators, Most of these comments were in support of the view that early use of calculators is detrimental to basic skill development. A further point made by a few was that some students cannot afford the graphing calculators required in high school courses, even though all schools have been provided with a set of these calculators for student use. We are not sure if this implies that there is a shortage of graphing calculators and the comments were not frequent enough to make a general statement about this.

The Conceptual/Investigative Approach. Respondents were divided in their opinions on the appropriateness of this approach. Some were clearly aware of its origin in the NCTM Standards and tended to be strong supporters of this approach. Others supported this in principle but raised many caveats about the time required for investigations, the language-dependence of the program, the lack of balance (implying a need for more attention to basic skills) and difficulties of implementing the approach in large or diverse classes or with low ability students. Many others were fundamentally opposed to the approach, mainly on the grounds that it neglects basic skills. It is fair to argue that the dominant view is that a rebalancing of the program is needed to ensure greater emphasis on basic skills in the early grades.

Teacher Capability and the Need for Professional Development. There was a tendency among respondents who were mathematics specialists to argue that many primary/elementary teachers are not comfortable with, or are even averse to mathematics. This argument typically led to further comments related to whether teachers can be expected to implement a conceptually-based program without having a deeper understanding of mathematics. On the other hand, even when probed on this matter, respondents were not prepared to argue that mathematics in Grades K-6 should be taught by specialists, as is typically the case for subjects such as music, physical education or French. There was ambivalence on this for Grades 7-9, with some believing that specialization is needed at this level.

The other extension of the teacher capability issue was frequent calls for more and better professional development. It was clear that those who had taken part in the recent summer institutes and even in the more common one-day workshops were positive about these experiences. There were comments on the high demand for summer institutes and for expansion of these activities. A few respondents demurred on this, suggesting that such institutes may not attract the teachers most in need of professional development.

Class Size and Diversity. Considering the high profile of class size as a public issue and the strong belief that smaller classes are more effective, we heard relatively few complaints about class size or comments that this is a significant factor in the state of mathematics teaching. Much more common were concerns about the wide range of student abilities in classrooms and especially about difficulties with functioning in classes with several students requiring modified courses or other accommodations. Class size arguments were almost always framed in these terms. We have heard this complaint in many other studies. It is clear that teachers are highly frustrated with the work involved in the development and implementation of Individual Student Service Plans and especially with course modifications. While few are ready to criticize the current inclusion model in principle, many feel that insufficient support is available for teachers in dealing with developmental delays and students with severe behavioural problems.

Specific to mathematics was a concern that low ability students cannot cope with the complexity of the current curriculum and that the requirement for explanation of the processes involved in arriving at answers is beyond the abilities of many students. The point made earlier about the high levels of language development required in the mathematics program was also related to this concern. Many teachers expressed the view that some lower ability students could do much better in mathematics if they could surmount the language barrier and that greater emphasis on basic skills rather than on explanation could allow some students success in mathematics. Others argued that a small amount or remediation would be valuable to students functioning on the margin but that the current model for allocation of special education teachers does not allow for remediation.

Homework. Although parents were not as widely represented in the focus groups as would have been desired, those who did attend echoed many of the concerns expressed by teachers. Homework was a particular issue with parents, with the same sentiments being expressed as in the submissions, namely that there is too much mathematics homework, that homework is being used as a way of getting through the program and that parents are having difficulty helping their children with what was commonly referred to as the “new math.”[9]

A few parents went beyond this and argued that parents should not be expected to become teachers or to do the work that schools cannot do. Many teachers, on the other hand, lamented the lack of parental involvement and expressed concern that students seem no longer willing to do homework. In this respect, though not in most others, the views of parents and teachers diverged.

[9] It is interesting to note that the term “new math” was originally applied to the mathematics programs of the 1960s, developed in response to the drill and practice approach of earlier years and first embodying ideas from the structure of the disciplines and constructivism. One gets the impression that the 1960s new math was replaced by something else in the generation in which many of today’s parents were in school (perhaps the 1980s in many cases) and that a new “new math” has emerged from the NCTM documents.

Time. The issues of too many outcomes and of homework are closely linked to concerns about time. Obviously, more outcomes could be addressed and greater depth achieved if more time was available. However almost no respondents took the next step to comment on how more time could be found. This point was not pressed strongly because any major changes in time allocation and use, such as lengthening the school year or day or increasing the time on mathematics at the expense of other areas of the curriculum have policy implications which go beyond the mandate of this study.[10] A few respondents spoke of the idea of integration across subjects in the primary grades, feeling that this could help save time.

The High School Program Structure. There were many comments on the current structure of the high school program, but there were few common threads in these comments. The most consistent view was that a more “practical” approach is required in the general mathematics sequence, oriented towards consumer and workplace needs. Most who spoke of this did not support the general concept that the desired outcomes are much the same for all students with the differentiation being mainly in depth of treatment. Some commented that although this was intended to facilitate movement across the levels, so as not to limit options, such movement rarely occurs and when it does, it is mainly downward from advanced to academic or from academic to general.

Similar views were expressed about the distinction between academic and advanced mathematics. Some respondents were of the view that the attempt to differentiate by depth of treatment means that nobody is doing truly advanced mathematics. Others felt that there is a need to return to separate academic and advanced courses in Grade 10.

[10] Department of Education officials will be aware that recent studies of time allocation and use have been done both within this province and for the Atlantic region. There seems to be no advantage in revisiting this issue at length here.

High school teachers in the focus groups tended to agree that too few students are taking advanced mathematics. However, as the next section shows, this was not the prevailing view in the survey (a difference that may reflect the selection of focus group participants). Teachers feel that this is linked to student perception that the academic course is less demanding and is likely to result in higher grades, with obvious advantages for post-secondary entrance and scholarships. There was little support for the current practice of integrating the Mathematics 3204 and 3205 public exams, allowing the higher mark to be used. While this point was not made explicitly, some appeared to hold the view that as long as academic mathematics is accepted for university entrance, there is little incentive to take the advanced courses.

A few respondents spoke of various ingenious ways schools have found to integrate or accelerate courses to accommodate two courses in one year. Regardless of other issues, it is evident that demand for mathematics is high and that most students are taking more courses than are needed for graduation. Post-secondary aspirations are likely driving much of this demand, perhaps offsetting any tendency to take easier ways out. For this reason, small schools have to find ways to offer a large suite of mathematics courses. This is commonly done by multi-course teaching in the same classroom or, in a limited number of cases, through on-line courses. Little was said about the latter in the focus groups and we did not press this issue because few participants had had first-hand experience with these courses.[11]

Several respondents took the position that the need for Mathematics 3103 and 3207 is evidence of failure to develop basic skills in the early grades and of general weakness in the program.

Impact of the CRTs. By all accounts, the CRTs are taken very seriously by the system and much work goes into preparation for these tests at Grades 3, 6 and 9. While a few would perhaps wish that the CRTs would just go away, the most frequent concern expressed was that preparing for and writing the tests takes time from instruction and that it is necessary to complete the mathematics program early in order to find this time. This exacerbates the problem of the crowded curriculum, making it more difficult to complete the expected outcomes.

[11] A full report on access and performance in on-line courses is in draft form through the CURA project at Memorial University. This should be available by the time this project is completed.

Scope and Sequence. There were a few comments to the effect that teachers are not aware of the full scope of the program and, in particular, they do not know what students have done in earlier grades and what they should expect students to know. Teachers argued that they introduce topics expecting that prerequisite material would have been done, but find that this is not so. A few commented that this may be related to the lengthy implementation period and the start at different levels at different times, and that it is only in the last year or so that all students would have been exposed to the full program.

Current Mathematics Initiatives. Respondents welcomed the *Excellence in Mathematics* initiative and certainly the infusion of funds that this is yielding. Positive comments were made about the role of the numeracy support teachers, the summer institutes and the curriculum review work which has resulted in some reduction and consolidation of outcomes. Some respondents urged patience in allowing the current program to mature and the impact of the new initiatives to be felt. However, the dominant view was that, despite these initiatives, significant curriculum change is needed.

Teacher Survey

Survey questionnaires were developed with reference points at Grades 3, 6 and 9, as well as senior high school teachers. The main focus of the surveys was on the mathematics curriculum. Questions were asked about breadth and depth of treatment, the value of the various curriculum documents and other resources available to support the mathematics program and the emphasis on specific outcomes. The high school questionnaire was somewhat broader, focusing on the program structure and courses, but not on specific outcomes.

The target populations consisted of all teachers who had taught the specified grade levels in the 2006-2007 school year. Details of the populations, samples and error rates, as well as a full summary of the results are given in a separate background report.

Questionnaires were sent to all schools with instructions to the principal to distribute to the appropriate teachers and to collect and return the questionnaires as a batch. Teachers were informed that their responses would be anonymous and were asked to place their completed questionnaire in a sealed envelope before returning to the principal. Questionnaires were distributed by the Department of Education but returned directly to the consultants. To help preserve anonymity, once the teacher envelopes were opened, these were separated from the school return envelope so that teachers could no longer be identified with their schools. No identifying information was included in the data files.

This section presents a summary of the main results from a preliminary analysis based on about 600 responses received up to the time of writing. A more detailed account of the survey is given in a separate background report.

- Only a small proportion (about 13%) of Grade 3 and Grade 6 (about 10%) teachers have mathematics as a major or area of concentration. The proportions are much higher for Grade 9 (72%) and senior high (83%).

- Nevertheless close to half of Grade 3 and 6 teachers consider themselves to be specialized in mathematics by virtue of their experience in teaching that subject. Almost all Grade 9 and senior high teachers place themselves in that category.

- Most Grade 3 and 6 teachers reported spending 40% or less of their time in teaching mathematics. The assignments of Grade 9 and senior high teachers are more highly concentrated with 56% and 70% respectively reporting that 70% or more of their teaching assignment is in mathematics.

- Most classes in Grades 3 and 6 have 24 students of fewer and about 40% have fewer than 20 students. Only 3% of classes in Grade 3 and 11% in Grade 6 have 30 students or more. Class size increases in the higher grades, with about 45% of classes having 25 or more students at both levels. 14% of classes in Grade 9 and 10% of those in senior high have 30 students or more.

- Almost all Grade 3, 6 and 9 teachers reported that they spend 50 minutes or more per day on mathematics. Nevertheless, the times vary considerably, with a few reporting less than 50 minutes and others as much as 80 minutes or more per day.

- Grade 3 and 6 teachers overwhelmingly (94% and 90% respectively) believe that the current mathematics curriculum is too broad. The same view is held by 69% of Grade 9 teachers.

- It might be expected from the focus groups and other information that a curriculum that is seen as too broad would also be seen as having insufficient depth. However, about half (53%) of Grade 3 and 43% of Grade 6 also believe that the curriculum treats the outcomes in too much depth. About 25% of both groups feel that the outcomes are not treated in sufficient depth. Taken together, the breadth and depth results suggest a curriculum that is viewed as far too intensive.

- In the case of Grade 9 teachers, 69% reported that the curriculum is too broad. However, close to three-fourths also considered the depth of treatment to be about right. This result is more consistent with the breadth versus depth arguments that we heard.

- The breadth versus depth issue is more complex and course-specific at the senior high school level. Teachers overwhelmingly believe (89%) that there are too many outcomes in Math 1204 and about half hold this view for the general courses; Math 1206, 2206 and 3206. For other courses most teachers believe the number of outcomes to be about right.

- For most courses, a large majority of teachers consider the depth of treatment to be about right. The exceptions are the general courses where close to half consider the level of treatment to be too deep.

- High school teachers were asked more specifically about the adequacy of time allocations for the various courses. With the exception of Math 1204, most teachers believe the time allocations to be about right. A large majority (87%) consider that there is insufficient time for Math 1204.

- High school teachers were also asked to rate the difficulty of the various courses. A majority considered most courses to be at about the right level of difficulty. Again, the general courses are the exception, with about half considering these courses to be too difficult.

- High school teachers overwhelmingly support the current three-level mathematics course structure at Levels II and III. About two-thirds support the propositions that three levels should also exist at Level I.

- A majority of high school teachers (61%) believe that the academic courses are inadequate preparation for some first year university courses. Despite this, high school teachers believe that the proportion of students taking advanced mathematics is about right.

- There is a division of opinion about the proportion taking the general courses, with close to half believing that this proportion is too high and half that is it about right.

- High school teachers are not strong supporters of distance education, with close to 90% taking the position that this should be used as a last resort, only when no classroom teacher is available and that distance education is appropriate mainly for the most highly motivated students.

- Large majorities of Grade 3 and 6 teachers are of the view that there is not enough emphasis on basic concepts and operations in the mathematics curriculum. Grade 9 teachers are more divided on this issue, with close to half in each case reporting the emphasis as about right (51%) or not enough (48%).

- About half of Grade 3 teachers reported using the prescribed textbook every or nearly every class. This number increases to more than 90% for Grade 6 and 9 teachers. The Curriculum Guide was also reported as being used almost every day by a majority of teachers at all three grade levels.

- The manipulative materials kits were reported as used every or nearly every day by about half of Grade 3 teachers. However, this falls to about 20% for Grade 6 teachers.

- Close to half of Grade 3 teachers reported that the textbook at that level needs a major overhaul. This response was much less prevalent at the Grade 6 and 9 levels, with most reporting that the texts are adequate but could be improved.

- Most teachers at these three grade levels feel that the Curriculum Guides and other resources provided by the Department of Education are adequate but could be improved.

- Very few high school teachers gave positive ratings to the textbooks in any of the courses. The exception was 3103, where the text was rated as excellent or good by 77%. Similarly, a majority gave fair to poor ratings for the Curriculum Guides, again with the exception of 3103, where most gave good ratings.

- High school teachers were asked to indicate their agreement or disagreement with a number of propositions about mathematics programs and students. The following are some results of major interest:

  - Almost all high school teachers believe that students are weak in basic mathematics concepts from earlier grades.
  - Most believe that students do not work hard enough and that students often take lower level courses than they should in order to get higher marks.
  - Teachers are divided on whether more students should take advanced mathematics but most disagree with the proposition that more students should take basic mathematics courses.
  - Teachers tend to disagree with the statement that schools tend to be too selective in assigning students to advanced mathematics.
  - A majority of teachers disagree with the proposition that academic mathematics is adequate to meet the requirements of first-year university mathematics courses
  - Most also disagree with the statement that university courses have too much influence on high school teaching.

Teachers of Grades 3, 6 and 9 were asked to rate the level of emphasis and the degree of student understanding of all the specific curriculum outcomes at the appropriate grade level. These results are complex and will not be reported in detail here. Only a few highlights are given.

- There are significant differences in the ratings of the different outcomes. At all grade levels, GCOs A and B (the numeracy strand) receive relatively high, though not always the highest ratings while GCOs E and G tend to be rated lower in emphasis than others. This suggests reduced emphasis on outcomes that are later in the sequence, which would be consistent with teachers following the sequence but not reaching the end. However, this pattern is not universal. For example, in Grade 3, the highest emphasis is on outcomes under GCO F (the uncertainty strand).

- In almost all cases, the ratings for high emphasis are considerably greater than those for high student understanding. This suggests that many teachers believe that students do not reach a high level of understanding on particular outcomes despite high emphasis on these outcomes.

- Nevertheless, ratings of student understanding were highly correlated with the emphasis ratings, suggesting that teachers believe that students understand better those outcomes which receive greatest emphasis. While this may seem obvious, an alternative possibility is that teachers might place more emphasis on outcomes that are more difficult for students to grasp.

An interesting feature of this survey is that a large proportion of respondents chose to offer comments in response to an open-ended question at the end of the survey form. This is unusual, as in most similar surveys we have done comments are fairly rare. A detailed summary of the comments is given in the background report. Most of the comments simply reiterated the responses given to the more specific questions. Thus, we again heard that there are too many outcomes, that there is insufficient emphasis on basic skills and that the textbooks are of poor quality.

A couple of points were made in the comments that were not addressed in the specific questions, but which reinforce what we heard in the submissions and focus groups. The first is that the strong emphasis on explanation and problem solving demands that mathematics students have high language skills and that those weak in language are at a disadvantage in mathematics also. Some went further, arguing that some students can do well in mathematics without high language skills and that these students need the reinforcement that might be available if they could show their mathematics skills independently of language.

The second major point is that some of the material in the current mathematics curriculum is not at a developmentally appropriate level for students at the grade levels at which the material is placed. This is an extension of the argument about the emphasis on conceptual development and suggests that teachers believe that their students are not ready for some of the deeper understandings of mathematics that the curriculum tries to promote.

V MATHEMATICS PERFORMANCE OF STUDENTS IN NEWFOUNDLAND AND LABRADOR

Although this study was more directly focused on curriculum than on student achievement, it may be argued that the main purpose of policies designed to improve teaching and learning is to improve achievement. The earlier comments on assessment and accountability relate to this point. While there may be other reasons for change, such as reducing cost or improving the working environment for teachers and students, the focus of accountability is on outcomes. This is clearly reflected in curriculum, where we have moved strongly in the direction of expressing curriculum in terms of what students should be expected to know or be able to do at successive levels in their schooling. The general, key stage and specific curriculum outcomes of the mathematics curriculum point in that direction. It follows that outcomes should be measured and the quality of the system judged in terms of the degree to which the stated outcomes have been achieved.

In this chapter, we summarize briefly the limited amount of available data which tell us something about the mathematics achievement of students in this province relative to those in other provinces and other parts of the world. Performance over time of students within the province is also examined. The latter should be of particular interest because if, as many seem to believe, the recent curriculum changes have led to a decline in quality of teaching and learning mathematics, then this should be reflected in the results over several years.

National and International Comparisons (SAIP and PISA)

The School Achievement Indicators Program (SAIP) is a pan-Canadian assessment of 13-year-old and 16-year-old students which was in place from 1993 to 2004.[12] During that time, three mathematics assessments were conducted, in 1994, 1997 and 2001.

Figures 4.1 and 4.2 give results for these three assessments for the majority language populations in each province (French in Quebec, English in all others). The bars represent the percentages of students meeting or exceeding the expected level of performance, Level 2 for 13-year-olds and Level 3 for 16-year-olds, on the SAIP 5-point scale.

Taking account of sampling error[13] in the comparisons, these graphs show that the performance of 13-year-old students in Newfoundland and Labrador was comparable to that for most other provinces with the exception of Quebec, Alberta and British Columbia. Performance did not change much over the three assessments. For 16 years olds, performance in this province was lower than that for most other jurisdictions, especially in 2001. Performance also declined over the period as it did for several other jurisdictions. Again, the highest performing jurisdictions were Quebec, Alberta and British Columbia. Nova Scotia was also fairly high in the first two assessments but declined significantly in 2001.

The Programme for International Student Assessment (PISA) 2000 assessment (Statistics Canada, 2001, 2004, 2007) assessed students in reading, mathematics and science. All of these assessments targeted 15-year-olds in all of the OECD countries as well as a number of partner countries. Mathematics was the major focus of the 2003 assessment. The mathematics results for that year are therefore more detailed and accurate than those for other years.

[12] SAIP has now been replaced by a new assessment called the pan-Canadian Assessment Program (PCAP). The first PCAP results will be available in early 2008.

[13] Sampling error is typically represented by “error bars” in the graphs. These have been removed here for clarity. However, it is important to note that differences of about ±7% are required to be able to state that one province or one assessment time is different from another.

Results for these assessments are shown in Figure 4.3. This chart shows Newfoundland and Labrador clustered among the other Atlantic provinces and Saskatchewan with average scores higher than the OECD average of 500 but lower than those for other Canadian jurisdictions. The Canada averages of 533, 532 and 527 were significantly higher than the OECD average. These were close to the highest among the participating countries. On the international comparisons in 2003, when mathematics was the major domain assessed, Alberta was second only to Hong Kong-China. In the same year, Newfoundland and Labrador was ranked 16th among 49 jurisdictions in comparing all countries and all Canadian provinces (treated as countries for the comparison). The Newfoundland ranking in 2006 changed only slightly, at 21st out of 57 countries. This is obviously a good result by international standards but remains low when the comparison is within Canada.

Figure 5.1 SAIP Mathematics: 13-Year-Olds

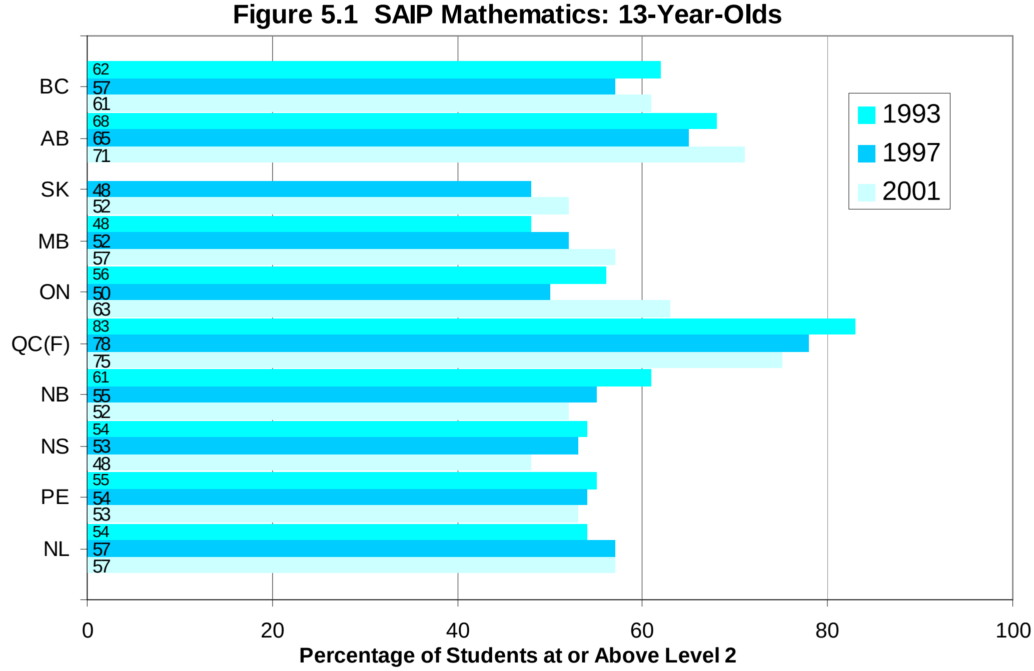


Figure 5.2 SAIP Mathematics: 16-Year-Olds

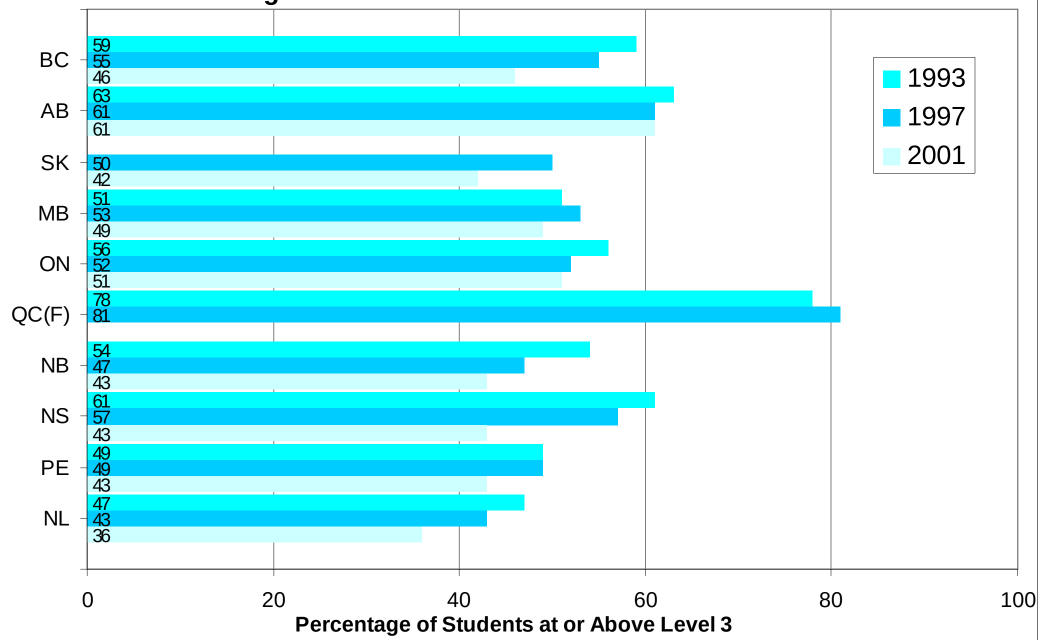
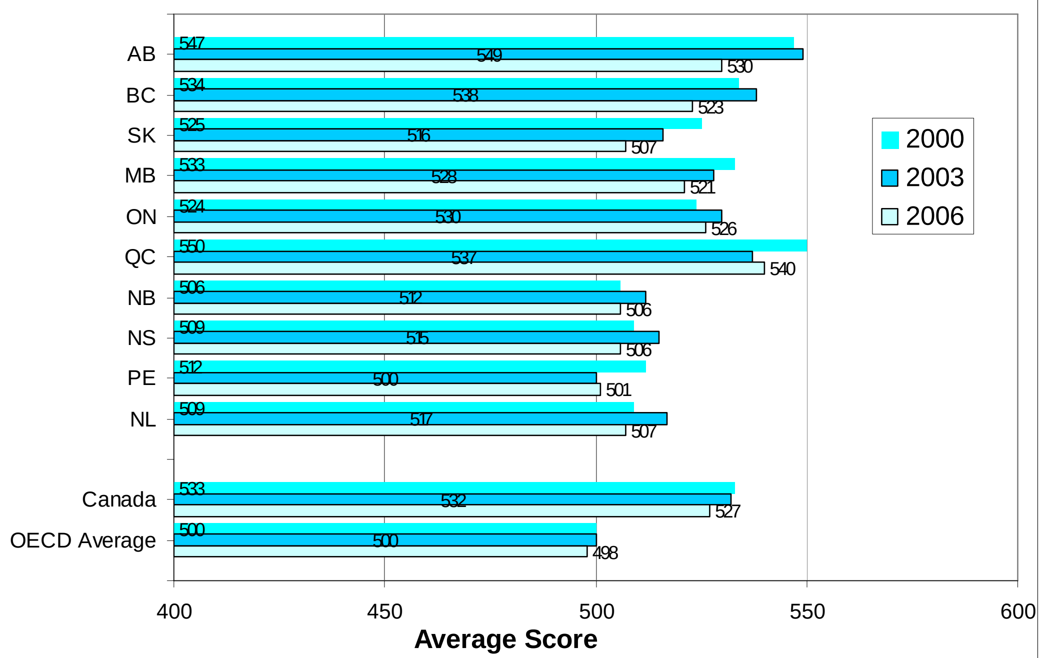


Figure 5.3 PISA Mathematics



Overall, these results do not point to any drastic deficit in mathematics achievement in this province relative to the highest performing jurisdictions but rather to a persistent low level difference that seems immune to change over time. Since most of the results predated the full implementation of the existing curriculum, and because so many other factors influence achievement, it is not possible to make any inference that these results are in any way related to curriculum change. Better indicators, based on local results over time, are needed to make this connection. Some of these indicators are summarized in the following sections.

University Mathematics Courses

The two large enrolment first year courses at Memorial University are now called Mathematics 1000 (Calculus) and Mathematics 1090 (Algebra and Trigonometry). Performance data for these courses were available from Fall1999-2000 to Winter, 2005-06. Mean scores and pass rates for these courses are presented in Figures 4.4 to 4.7. The Fall Semester results are of greater interest than those for Winter because most new high school graduates find their way into these courses in Fall.

It is evident from these graphs that a substantial drop in success occurred in 2000 and 2001 for Math 1000 and in 2000 for Math 1090. Since that time, Fall mean scores and pass rates have improved more or less continuously. Math 1000 success rates remain lower than they were in the 1999 baseline year. However, those in Math 1090 have improved significantly. It is interesting to note that these success rates are significantly higher than those of the late 1980s which triggered the last major program review (Crocker, 1989).

While this is an encouraging trend, these success rates are significantly lower than those found in most other first year university courses, suggesting that mathematics remains a struggle for many students. It is also important to note that there may be many reasons for these changes that are unrelated to the state of high school mathematics. Selectivity differences, changes in the composition of the student body, or changes in course content or grading practices are a few examples. We have not been able to examine these factors. However, it is interesting to note that enrolments in these courses have remained quite high, and that university participation rates among high school graduates are also high.

Figure 5.4 Mean Score Mathematics 1000

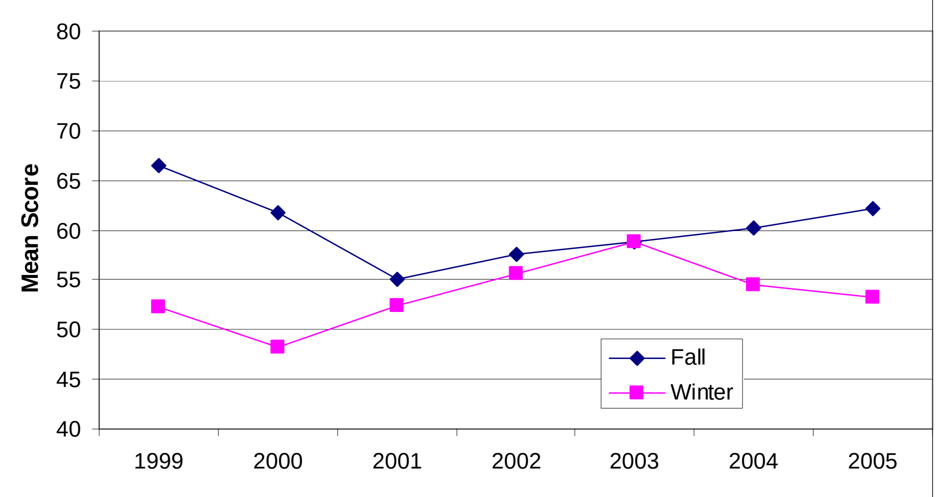
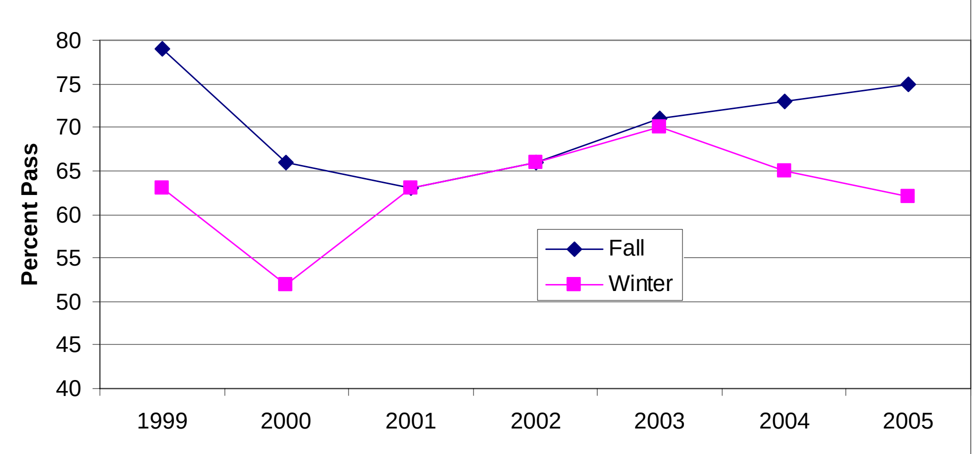
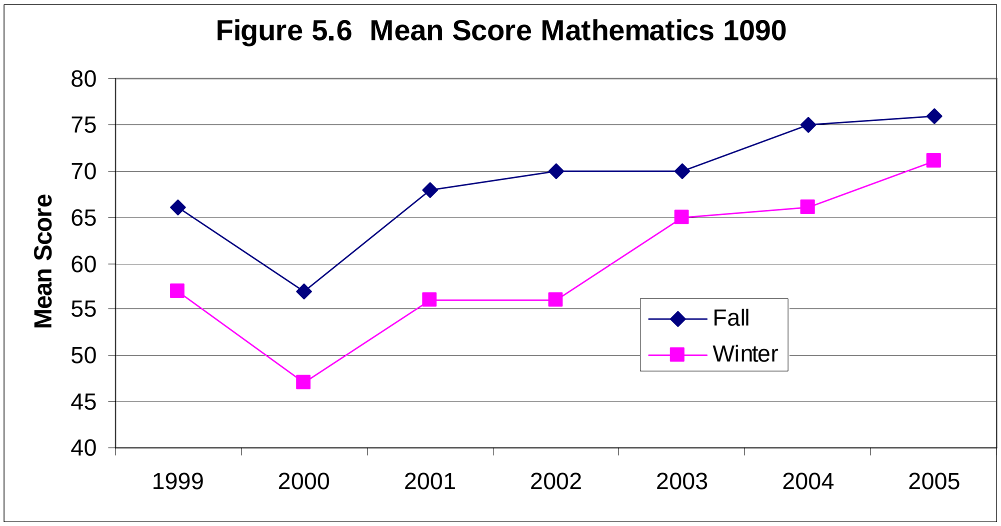
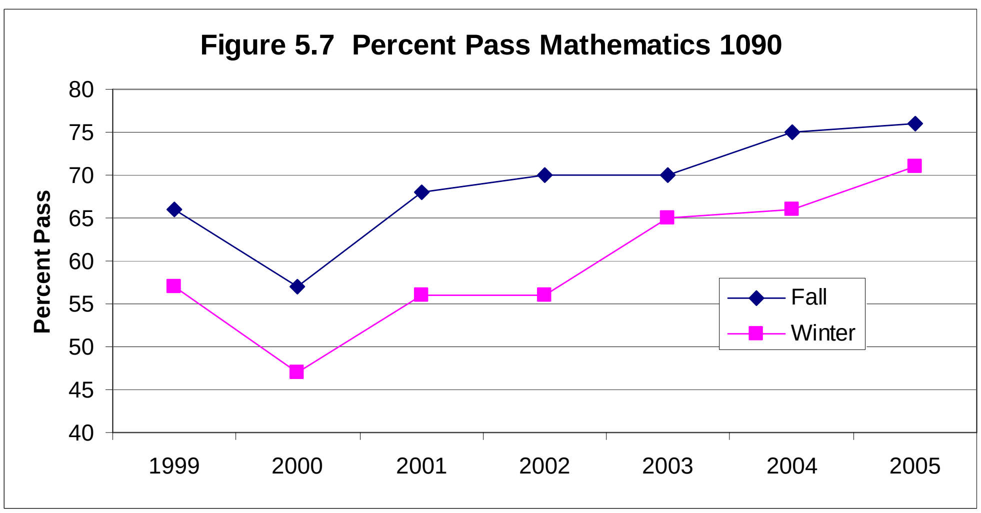


Figure 5.5 Percent Pass Mathematics 1000







High School Grades

Results for school and public examinations in the two final year mathematics courses are presented in Figure 4.8. The main focus here should be on the public examination results, both because data are available for more years and because this measure is common to all students in a course.

Aside from a couple of anomalies, the pattern here is one of gradual improvement in performance over the past few years. Since this coincides with introduction of the new mathematics program, the straightforward inference is that the new program has led to improved performance. However, this would be true only if there had been no significant changes in the test, the grading scheme or the population of students taking the test. As far as we can determine, all of these features have been reasonably stable in recent years, making it plausible to argue that there has been some improvement in performance. However, further analysis would be required to rule out all other sources of change. For example, it remains possible that teachers and students are becoming more accustomed to the new curriculum and are adapting their approaches so that little is lost. The existence of the supplementary courses, 3207 and 3103 may be significant in this respect, although this is difficult to demonstrate empirically.

Aside from the change over time, the most striking thing about these numbers is the extremely high success rates in Mathematics 3205. Means for both public and school are substantially higher than for Mathematics 3204 and the pass rates are approaching 100%. Of course, it might be argued that this course attracts the best students and one might therefore expect grades to be higher. However, it is just as plausible to argue that the course should be more difficult than 3204, which should bring marks closer to those in 3204. If 3205 is not more difficult, but simply different, there would seem to be no point in restricting this course to the best students. While 3204 and 3205 are similar in content, the intent is that 3205 have greater depth of treatment which, in some obvious sense, ought to make it more difficult.

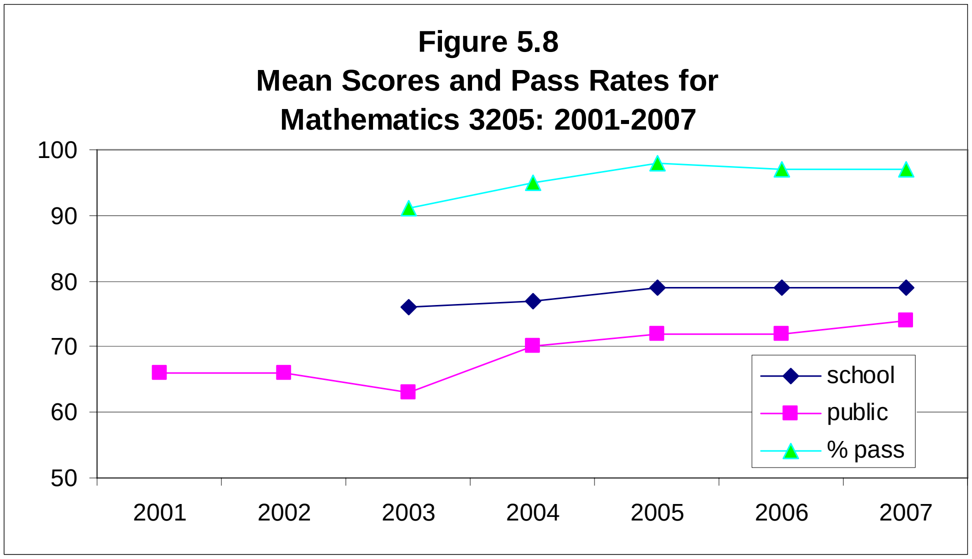


Figure 5.8
Mean Scores and Pass Rates for
Mathematics 3205: 2001-2007
100
90
80
70
60
50
2001
2002
2003
2004
2005
2006
2007
school
public
% pass

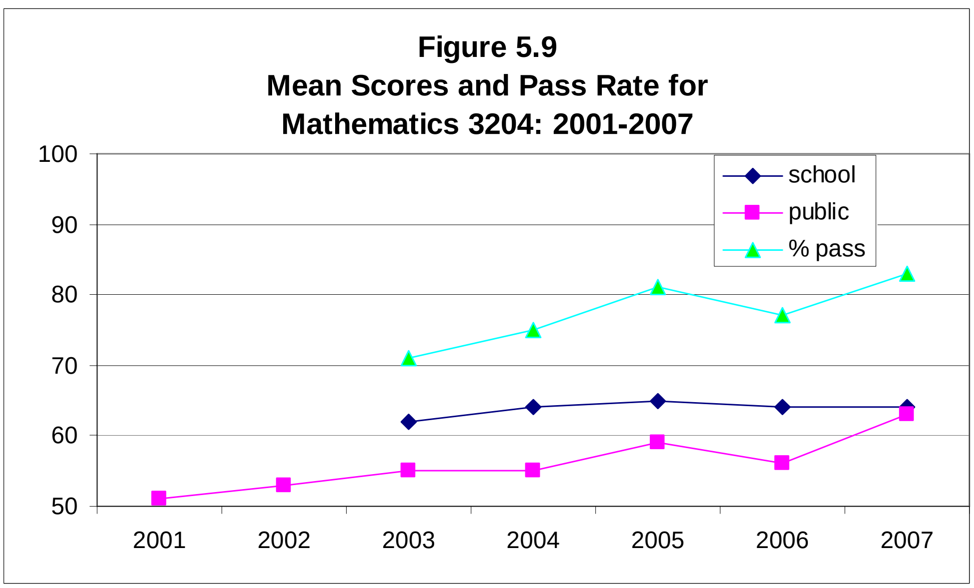


Figure 5.9
Mean Scores and Pass Rate for
Mathematics 3204: 2001-2007
100
90
80
70
60
50
2001
2002
2003
2004
2005
2006
2007
school
public
% pass

While there is no way to demonstrate that individual students would do as well in 3205 as in 3204, it is obvious that significantly larger numbers (of lower ability students) would be needed in 3205 before the average grade was as low as in 3204. An argument can be made that taking advanced mathematics conveys significant advantage for university work. That would likely be true even for students who may get lower marks than they would by defaulting to 3204. The problem is that lower mathematics marks affects student averages and hence their prospects for post-secondary admission and scholarship eligibility.

The conclusion we draw from this is that 3205 is much more selective than it needs to be. We have heard from many sources that students are selecting the academic rather than the advanced course because it is seen as easier, yielding higher marks that can help with university admission or scholarships. On the other hand, the survey suggests that most teachers believe the proportion of students taking advanced mathematics to be about right and that schools are not too selective in placing students into the advanced program. It is difficult to reconcile the observed results with these teacher beliefs and it may be equally difficult to persuade schools and teachers to admit more students to the advanced courses.

It is noted that the Math 3205 exam is now divided into sub-scores, allowing Math 3205 students to obtain a mark for both 3204 and 3205. The 3204 score is based on a sub-set of the total exam. The 3205 score is combined with the school grade to yield a final result for that course, while the 3204 score is used alone. Presumably if the latter is higher, this grade can be used in computing averages for scholarships or post-secondary admission. We have not analyzed the effects of this change. However, this might provide an opening to give many more students an opportunity to take 3205. Essentially, there seems to be nothing to lose in marks but perhaps much to gain in mathematics capability by taking 3205.

We had hoped to do a more complete analysis of the relationship between high school and university performance, and actually have access to data which would allow this. However, the matching of data files proved to be more complex than anticipated because students do not always take either the high school or relevant university courses in the same year. Tracking all students entering university in one year would therefore require reaching back to earlier years in the high school program. While possible, this is quite time-consuming. Nevertheless, preliminary analysis indicates the following:

- Most students taking Math 3204 in high school take Math 1090 in university, while most taking Math 3205 and/or Math 3207 take Math 1000. We understand that the university discourages those with marks less than 80 in Math 3204 from taking Math 1000. These students are encouraged to take one of several foundation courses available through the Mathematics Learning Centre.

- There is a high correlation between high school and university mathematics marks. This is highest for Math 3205 and 3207 for students taking Math 1000 at university. It is lower for Math 3204 students taking Math 1090.

- School, public exam and combined marks in Math 3204 and 3205 have about equal predictive value for marks in Math 1000 and 1090.

- Grades in university courses are considerably lower than those in high school courses. Students with a final grade of less than 70% in Math 3204 have considerably less than a 50% probability of passing Math 1090. On the other hand, those with a final mark of 80% or more had better than a 90% chance of passing.

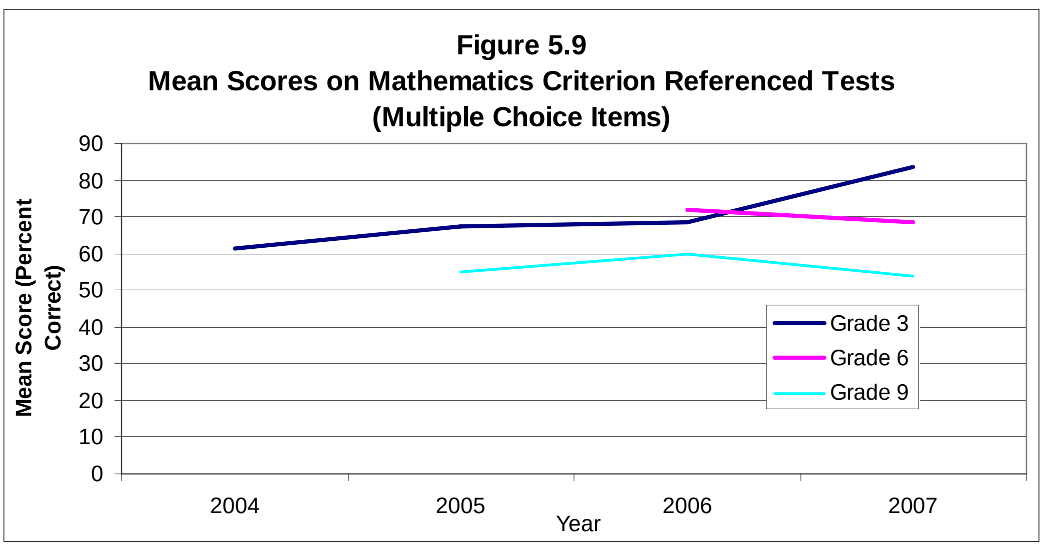
- The situation is somewhat better for those from Math 3207 who take Math 1000. Those with Math 3207 marks in the 60-69% range show a considerably greater than 50% probability of passing while almost all of those with a Math 3207 mark of 80 or more can pass Math 1000.

Criterion Referenced Test Results

Criterion Referenced Tests (CRTs) in English language arts and mathematics are now administered to all students in Grades 3, 6 and 9 each year. These tests consist of both multiple choice and open ended items. However, since this is a recent innovation, limited time series data are available. In particular, there is no way to compare performance on a standard scale over a time line which extends back to the previous curriculum.

The available results for the mathematics multiple choice items are shown in Figure 4.9.[14] These results show a pattern of improvement for Grade 3 since 2004, with the gain particularly evident in 2007. A slight decline at both Grade 6 and Grade 9 can be seen from 2006 to 2007. However, these results are too limited to draw any conclusion about the trend. The interesting pattern to watch will be the performance at Grades 6 and 9 of students who seem to have done so well in Grade 3 in 2007.

Grade 3 students have been exposed to the current curriculum for the whole period covered by the test results. We understand also that there have been no significant changes to the test during this period. There are certainly no changes in the way that multiple choice items are scored. It is also unlikely that the overall student population has changed significantly over the past few years. Many of the potentially confounding factors have thus been removed. However, we do not know if schools or teachers have done anything that might have led to improved scores. Certainly the tests have come to be taken as a marker for school performance and it is possible that poorer performing schools in the earlier years have done things to improve scores. In the absence of detailed knowledge of what is happening in the schools, we must caution against drawing any direct inference that the improvement in Grade 3 is directly linked to the curriculum.



[14] The results for the open ended items are more limited and not readily expressed as means, so these are not presented here.

VI SUMMARY, ANALYSIS AND RECOMMENDATIONS

Summary

The main findings of the study may be summarized in the following points:

- The literature seems to support the idea that mathematics should be taught for conceptual understanding and that this does not interfere with basic skill development. However, it is not at all clear that this view enjoys public or teacher support.

- Although there is some support in principle for the conceptual/investigative approach to mathematics, there is strong evidence that teachers feel that the balance has shifted too far and that students are progressing through the grades without having mastered basic skills needed as more advanced mathematics content is introduced. Evidence from parents is more limited but the submissions and focus groups clearly support the teacher view.

- More specifically, there is a strong view on the part of teachers that the mathematics curriculum is too crowded and that it is impossible in most grades to cover all of the expected outcomes in the depth required for adequate student learning.

- A second major issue is a perceived lack of match between outcomes and textbooks and a perception that many of the prescribed texts are of poor quality.

- The comparative curriculum analysis revealed that the total number of outcomes included in the local mathematics curriculum is not very different from the totals found in contemporary programs in other jurisdictions. If the local mathematics curriculum has too many outcomes that flaw is shared by other jurisdictions in Canada.

- However, studies in Western Canada have shown similar concerns to those expressed in this study. A new *Common Curriculum Framework* developed by the Western and Northern Canadian Protocol (WNCP, 2006) contains many fewer outcomes and shifts the balance, especially in the early grades, to the numeracy strand. This framework will form the basis for curriculum revision in the four Western provinces and in the territories over the next few years,

- The revised version of the *NCTM Principles and Standards* addresses part of the problem of balance. The *Curriculum Focal Points* document published in 2006 makes it clearer that some areas have higher priority than others and more explicitly identifies priorities in basic skill areas. The new WNCP curriculum framework though predating the *Focal Points,* is consistent with this direction.

- The combination of a crowded curriculum and diverse student needs and abilities seems to make the working lives of many teachers highly complex and stressful. While this is not confined to mathematics, we were given the distinct impression that mathematics is one significant source of this complexity. Unfortunately, we were unable to explore this phenomenon in the detail needed to draw strong conclusions about teacher workload and its impact on learning.

- The available comparative achievement evidence shows that students in this province perform above the international average but exhibit slight and persistent lower performance relative to the Canadian average and to the highest-performing provinces. On local measures, most of the evidence points to improvement in recent years. However, it is not possible to attribute any of these results directly to the curriculum. Many other factors are known to have small cumulative effects on achievement and curriculum is not likely to be the decisive factor.

- There was concern that teachers have not received adequate professional development to deal with the current curriculum. The view was expressed by some that primary/elementary teachers are often not comfortable with mathematics. This was supported to some extent by the survey data.[15] It is not clear if this problem can be solved by professional development. However, it is clear that more intensive PD efforts are required if the principles underlying the new mathematics program are to be maintained. This is especially so if further significant revisions are made.

- There is strong support for program differentiation at the high school level but there are varying views on its extent and on how far down the grade levels this should extend. We heard calls for having a choice of courses in mathematics as early as Grade 5 and other more cautionary views for not closing doors to mathematics too early. A majority of high school teachers feel that the three-level course structure should be extended downward to Level I (Grade 10).

- Many teachers are highly frustrated with having to deal with ISSP development and implementation and, more generally, with the disruption of normal classroom activity caused by the presence of high-needs students or those with severe behaviour problems. Few are critical of the inclusion approach in principle but feel that the system is insufficiently resourced to be successful. Investigating these issues was not part of the mandate of this study. The issue is raised because it illustrates how one aspect of school and classroom activities can impact on others.

[15] In retrospect, the professional development issue should have been pursued in more detail in the survey. It was not anticipated at the beginning of the study that this would be as much of an issue as it proved to be. An upcoming study of the intermediate program offers an opportunity to pursue this issue in more depth.

Curriculum Development

Based on the comparative analysis and the consultations, it is difficult to escape the conclusion that the mathematics curriculum in this province is unsustainable in its current form. A successful curriculum must enjoy the support of the public and certainly of the teachers who must implement it. It is clear that such support does not exist. Even accepting in principle the conceptual/investigative approach embodied in the NCTM documents, public and teacher views about the neglect of basic skills must be considered. The obvious conclusion is that there is a need for rebalancing the curriculum to respond to the calls for greater emphasis on basic skills, while not losing the gains that have been made in developing conceptual emphasis

Teachers' views that there are too many outcomes are related to the issue of basic skills. Short of increasing the already substantial time devoted to mathematics, the only way to rebalance the curriculum is by reducing the total number of outcomes expected. This requires that some strands or sub-strands be consolidated, given lower priority or removed entirely from the curriculum. Indeed, this has been implicitly recognized by the Department of Education through its recent modifications to the outcomes. The question is whether continued refinement of this nature will suffice or whether a more radical overhaul of the curriculum is needed.

Our general conclusion is that the problem of too many outcomes stems mainly from an interpretation of the 1989 NCTM *Standard* which says that equal emphasis on all strands is required at all levels (the only exception seems to be the uncertainty strand in the early grades). This "spiral approach" results in the repetition of outcomes, at slightly greater levels of depth, in successive grades. The consequence is introduction of some outcomes earlier than necessary, with cursory treatment of these outcomes because children are not ready for their detailed treatment.

In publishing its *Curriculum Focal Points (2006)*, the NCTM seems to have recognized this and more explicitly established priorities on two strands, numeracy and geometry, in the early grades. Emphasis on these is gradually reduced in favour of algebra in the higher grades. The remaining strands receive relatively lower emphasis throughout. This approach seems consistent with what is being requested in the field. NCTM seems also to have come to the conclusion that its *Standards* are being misinterpreted as advocating neglect of basic skills. The *Focal Points* are an attempt to establish priorities and thus amount to an admission that too much is being attempted.

As it happens the *Common Curriculum Framework* being developed under the WNCP addresses exactly this problem. Although this retains a strong emphasis on conceptual mathematics, it also brings back a basic skills agenda, accomplishing this by a significant reduction in the total number of outcomes, especially in strands other than numeracy in the early grades. For this reason, and also for reasons of efficiency and implementation time, our core argument is that Newfoundland and Labrador should adopt the WNCP framework in revising the mathematics curriculum, rather than embarking on another protracted curriculum development effort.

The second major source of frustration with the existing curriculum is the perceived low quality of the textbooks, the lack of match of the texts to the curriculum guides and the lack of appropriate practice exercises in the texts. The concern relates to the centrality of the textbook in the minds of teachers, students and parents. While some would argue that the textbook is only one of many resources, the reality is that the text is the only document in the hands of all students and available to parents. It is futile to argue that the textbook should be downplayed as one of many resources, as long as this is the only resource available to all.

The quality of the texts is related to another aspect of the situation, that of teacher workload. The main issue on the part of teachers is the amount of effort required to locate suitable instructional material. Again, the conventional wisdom seems to be that teachers should be responsible for developing their own approaches and their own materials, responding to the uniqueness of their students and classes. However, teachers are strongly of the view that materials should be readily available and that developing materials to fill gaps in the curriculum adds to an already high workload. This is exacerbated when course modifications or other accommodations have to be made for ISSP students.

Some teachers drew attention to the fact that a few of their colleagues have been preparing and selling materials to supplement the mathematics program. Others raised the same point about a growing tutoring industry. The cost of these supplementary materials has become an issue, with reports that students are being urged to buy these, even as textbooks are now fully subsidized. The idea that subsidized resources are viewed as so inadequate as to engender "back door" costs for supplementary materials is rather disconcerting and defeats the purpose of the government's free textbook policy. If the local materials being prepared are better than the prescribed texts, perhaps these are what should be subsidized. Unfortunately, not having anticipated this issue, we did not address the extent of this problem in our survey. In any event, recommendations intended to address this problem are to be made in this report.

Our own view is that mathematics has few elements that are unique to local schools or students and that there should be no need for teachers to be extensively involved in locating and preparing their own mathematics materials. Curriculum guides and textbooks, as well as supplementary materials such as assessment guides, manipulatives, practice exercises, policies on the use of calculators and such should be comprehensive enough to form a complete package. While this does not address the issue of course modification or accommodations for special needs students, the latter might be less of a burden if the available materials were considered adequate.

The obvious question is "where do we go from here?" While some have urged patience in allowing the "new" curriculum to mature, it is clear that other jurisdictions are now well along in developing what may be called a "second generation" program, based on the 2000 NCTM *Principles and Standards* and the *Focal Points*. There seems to be little point in waiting several more years before starting down that road. Indeed, even if we started now, several more years would be required, at the normal curriculum development pace, before a revision would emerge, especially if the existing consortium arrangement is continued, as the work would require coordination among the four Atlantic Provinces. In the meantime, the unsatisfactory existing situation would remain. Energy that might be spent on further tinkering with the current program would be better spent on moving as rapidly as possible to a second generation program. Fortunately, there is a way to move in that direction with comparatively little effort or cost.

The Western and Northern jurisdictions have already moved almost exactly in the direction that this review would recommend. Therefore, the obvious way to get rapidly to the next stage would be to adopt the WNCP 2006 and 2007 Common Curriculum Frameworks as the basis for development. There is no point in initiating a new local or regional initiative designed to get us to essentially the same place. The effort and cost that this would entail would be better directed at implementation than at development. It is therefore recommended:

Recommendation 1

That the WNCP Common Curriculum Frameworks for Mathematics K-9 and Mathematics 10-12 (WNCP, 2006 and 2007) be adopted as the basis for the K-12 mathematics curriculum in this province.

It is worth noting that these framework documents are comprehensive, including a conceptual framework derived from the NCTM *Principles and Standards* (2000), general and specific curriculum outcomes and a new component called "achievement indicators." We believe that the latter is an important innovation because it gives specific indicators of whether the student has met the outcome. The achievement indicators also provide a guide to assessment, both at the classroom level and for provincial assessments.

Some may argue that full adoption of a program developed elsewhere, rather than adapting this program to local circumstances, fails to address unique features of schools, teachers and students in this province. We would support this perspective if the subject at issue were social studies, music, or even aspects of language arts, where local culture is an important component. However, mathematics is arguably the most universal of school subjects, and the one for which the performance expectations at higher levels of education, in the workplace and in society at large are the same no matter where one is located.

That is not to say that there are no unique challenges in implementing mathematics curriculum in the local system. The next couple of sections examine issues of implementation.

Implementation Schedule

The current curriculum took about four years to develop and six years, from 1999 to 2005 to be fully implemented. It is not surprising that in that decade, new curriculum thrusts would have emerged. While it would be an exaggeration to say that the current curriculum was obsolete before it was implemented, the lesson here is clearly that we should not wait another decade to see changes, that are being called for locally and that are emerging elsewhere, to be implemented. Part of the motivation for recommending adoption of an existing framework rather than further developmental work is to help ensure that the next round of change occurs on a faster track.

Even assuming that the recommendations to adopt the WNCP 2006 and 2007 framework documents is accepted immediately, there remains the problem of the schedule for phasing in a new program. Strict adherence to the idea that mathematics is cumulative would seem to preclude full implementation in one year. On the other hand, what is being proposed does not represent such a dramatic departure from the existing curriculum that we should assume that students at a given grade level would be completely unprepared for what is to come. Indeed, if the current problems centre mainly around basic skills, that problem will remain even if no changes are made. There is little in the proposed curriculum that is not already found somewhere in the existing one. The main change is in the direction of reduction in outcomes and greater depth of treatment. This suggests that there may be some room in a new program for remediation where necessary; something which would be desirable even under the existing curriculum.

It therefore seems to make sense to work towards rapid implementation. The Western jurisdictions are working on a schedule which would see the K-9 program implemented over three years, beginning in 2007. As we understand it, resources are not yet ready for the high school program, so its implementation has been scheduled to begin in 2010 in Alberta and British Columbia and 2009 in Manitoba. Following the same schedule as is now in place in Alberta and British Columbia, but with a one-year delay, the following schedule is recommended for K-9:

Recommendation 2

That implementation commence with Grades K, 1, 4, 7 in September, 2008, followed by in Grades 2,5,8 in 2009 and Grades 3,6,9 in September 2010.

Of course, there may be some obstacles to implementation on this rapid schedule. First, some negotiations would likely have to take place to acquire the program or to become a participant in the WNCP (leading to a WNNLCP perhaps?). The copyright statement on the WNCP documents indicates that they may be copied for educational use, so simple adoption is technically not a problem. However, we did not inquire about the views that the Consortium may have on another province adopting without becoming an active participant. Some discussion at senior official levels would be appropriate to determine this.

The implementation schedule is contingent on the ability to put in place the necessary funding, especially for textbooks but also for professional development and other implementation activities. A one-year delay in this schedule would not be a disaster, but further delay risks getting further behind in the cycle and would likely require further "patches" to the current program.

Recommendation 3

That the senior high school program be implemented on the same schedule as now proposed for the Western and Northern jurisdictions, starting in 2010.

Although this seems to be a longer than necessary delay, we understand that support materials for the WNCP 2007 framework, particularly textbooks, are not yet ready. Also, there is less urgency at this level than at the lower levels. Though there are concerns at the post-secondary level about lack of preparedness of students entering from high school, success rates have not notably declined in recent years.[16] The main issues at the high school level are clearer differentiation of the basic, academic and advanced levels and improvement in most of the textbooks. The current add-on courses (3207 and 3103) will continue to be needed over this period in any case. However, there should be no reason to continue these after the new program is implemented. Courses such as these are not found in other jurisdictions.

[16] That is not to say that we judge success rates in first-year mathematics to be satisfactory. These remain lower than in other post-secondary subject areas. However, we have not been

The proposed implementation schedule is summarized in the following table:

2008	2009	2010	2011	2012
K, 1, 4, 7	2, 5, 8	3, 6, 9, 10	11	12

It has been standard practice in this province to pilot new programs before they are fully implemented. However, there are strong arguments for not piloting in this case. First, the proposed program is not based on a significant change in the overall framework. The rebalancing towards greater emphasis on basic skills is almost universally supported and represents familiar territory for most teachers in any case. The jurisdictions under the WNCP are not piloting the program. However, some are allowing voluntary implementation for one year. The disadvantage of this is that it requires two programs to operate in parallel, with accompanying complexities in supply of resources, professional development and assessment. The one year delay relative to other jurisdictions should give some room to determine if major problems emerge. Finally, we have rarely seen a pilot project that was unsuccessful. Pilots tell us less than we might think about difficulties with a program because pilot teachers tend to be enthusiasts, capable of making any program work. Only when the program goes to full scale operation are many of its flaws detected.

Recommendation 4

That the proposed revised program not be piloted but that an effort be made to learn from the initial experiences in other jurisdictions implementing the program in 2007-08.

able to detect a trend to indicate that success has deteriorated under the existing mathematics program.

Textbooks

Aside from breadth of the program, the quality of textbooks was the most frequently expressed concern about the existing program. The major source of teacher concern was the lack of match between textbooks and curriculum guides. We heard arguments that this is due to the high cost of textbook changes, requiring that new programs be implemented while retaining older texts and the fact that the local market is not large enough to attract publishers to produce customized texts for local programs.

The text is considered by most teachers and parents to be the most crucial resource. Unsatisfactory texts add to teacher workload and parent frustration and result in a market for supplementary material, with its attendant added costs. In mathematics, there is no compelling academic reason why there should not be a complete match of the text to the curriculum. Despite the high cost of text replacement, it is unlikely that any revision to the curriculum will be considered satisfactory without texts which match the new program.

The problem of market size effectively resolves itself if, on adoption of the WNCP frameworks, the accompanying texts and other resources are also adopted. Publishers have already produced texts that are compatible with the K-9 framework and we understand that the same is being done for the 10-12 framework. Cost aside, the obvious solution is to adopt these texts along with the frameworks. Since we were not asked to estimate costs, this has not been done. However, we anticipate that the Department of Education will have to devote some effort to the cost issue.

Recommendation 5

That textbooks and other resources specifically designed to match the WNCP frameworks be adopted as an integral part of the proposed program change.

Although more than one publisher had entered the business of producing texts compatible with WNCP 2006 and 2007, we have not examined these texts in sufficient detail to make a specific recommendation. Indeed, all such texts should be very similar in content. Closer examination may reveal differences in format, supports available or costs that may help determine the choice.

Student Achievement

Although there seems to be a widespread belief that the current curriculum is leading to a decline in student mathematical capability, there is virtually no hard evidence to this effect. Scores on both public exams and in university courses are actually gradually improving. Nevertheless, some would argue that the public exam results merely reflect the curriculum, and that improved scores are not necessarily an indicator of improved mathematics achievement except within the narrow context of the courses being examined. Similarly, it may be argued that students taking first year university mathematics courses in recent years were not exposed to the new curriculum but, if the worst fears of the critics are realized, we can expect a deterioration in performance on the part of the next generation of students.

It is not possible to determine if improved scores on local measures is due to the curriculum itself, changes in expectations, the compensatory efforts being exerted (e.g. Mathematics 3103 and 3207 or activities such as tutoring or the Mathematics Learning Centre at MUN) or other factors. Indeed, so many factors can exert small but cumulative influences on learning over a student's career that it seems unlikely that the structure of the curriculum would be the decisive factor. The case for curriculum change, as argued in the previous sections, is not made on the basis of evidence on achievement but rather on comparative evidence and an evolution in the fundamental thinking about what should constitute an appropriate mathematics program. The case is reinforced by the consultative evidence, which clearly indicates that almost none of the important stakeholders are content with the current program.

Although the local evidence on achievement trends is positive, the national and international comparative results indicate that we are in, at best, a static situation. The standing of students in this province has not improved significantly relative to other jurisdictions. While some might argue that being at a level comparable to most other Canadian provinces and at a slightly better than average international level is satisfactory, an argument can be made that maintaining the status quo should not be the goal. An *Excellence in Mathematics Strategy* should have excellence, not continued mid-range performance, as its target.

In any event, it is difficult to argue that current success rates in first year university mathematics are satisfactory, even if they are improving. Without wishing to over-emphasize the importance of university performance, it must be recognized that large numbers of students aspire to attend university, and actually do so. Success in first year mathematics opens up so many opportunities for further education that that goal should be for almost all students to succeed mathematics, just as they do in most other first year courses.

The 1989 Mathematics/Science Task Force Report took the need to improve achievement levels in mathematics and science its core argument, stating that the time had come to create a culture of high expectations. In 1994, following the Royal Commission Report (Government of Newfoundland and Labrador, 1992), the government produced a blueprint for improving learning (Government of Newfoundland and Labrador, 1994). The declared goal was to bring about a substantial improvement in the quality of education, to the point where the achievement of our students would rank with the best in Canada. This goal is based on the belief that high levels of education are essential to the economic and social well-being of the province. In our view, this goal has not yet been achieved. Now, however, there is evidence that the government and the public are ready to set higher targets for achievement and to implement measures to accomplish these targets. We take the *Excellence in Mathematics Strategy* and other components of the government's educational policy as evidence that this is the case.

This therefore seems to be the time to reiterate the need to create a culture of high achievement and to restate the more specific target of bringing the achievement of students in this province to a level comparable to the best in Canada. The immediate target for this is mathematics, although the goal is applicable across the whole curriculum.

Professional Development

A significant amount of professional development (PD) work has been done in support of the current program. Despite this, there were calls for more PD activity. Concerns were also expressed about the effectiveness of the one-day workshop approach to PD, something that we have heard many times before. There is substantial support for the summer institutes that have been conducted in recent years, and demand for these institutes has greatly exceeded capacity. The recent hiring of a significant number of numeracy support teachers is also viewed as a positive move, although there are arguments that these personnel are already spread very thinly.

The professional development work already done is not negated by the proposed changes. Although adoption of a new framework means many changes in detail, the fundamental approach remains grounded in the NCTM *Principles and Standards.* The major problem is not the need to start over in mathematics PD but that the current PD model is not seen as particularly effective. Teachers have complained that they have never received any PD for the new mathematics program, that the PD that has been available comes too late, and that there is a lack of follow-up of the typical workshop.

Unfortunately, the literature offers virtually no hard evidence on the effectiveness of any particular approach to PD. The value of PD in support of program change is taken largely on faith, under a common sense notion that teachers need to be brought up to date with intended changes. If the literature offers any guidance it is that sustained effort is needed, extending beyond any initial formal sessions. It seems to be virtually impossible for PD designed to support program change to have any significant impact unless it becomes much more intensive than anything that can be done under the current approach.

The recommendations which follow are intended to support this common sense approach, adapting the general idea of more intensive and sustained PD to local conditions. However, they are also intended to encourage a shift in thinking about the operation of PD activities. In particular, the thrust of the recommendations is to move to a model which encourages intensive PD for all teachers who are expected to implement new programs, a shift to a school-based model for program implementation and measures to sustain the knowledge and skill acquired in any intensive sessions.

In the short term, accompanying the proposed program revisions, what is required is a relatively intensive program designed to reach all teachers at the grade levels in which the program is being introduced. Even if minimally effective, this has the advantage of familiarizing teachers with the program and heading off complaints about lack of PD. Assuming a Grade K, 1, 4, 7 implementation scenario in 2008, that would require that something be done late in the 2007-08 school year, in Summer, 2008 or at the beginning of the school year in 2008. This would have to be done for close to 1,000 teachers.

There are three main ways in which PD can be implemented; summer institutes, school shutdowns and use of substitute teachers. Each of these has well known advantages and drawbacks. School shutdowns would involve relatively little direct cost, especially if the session is school-based, with no teacher travel required. However, closing a school detracts from instruction and, in this case, would involve more than just the mathematics teachers in the school. This might work reasonably well for primary/elementary grades, where classroom teachers teach mathematics and where most other teachers might also find something useful in a mathematics PD session (or find something else to do at the same time). However, this would be less appropriate for high schools where most teachers are not involved in mathematics.

The use of substitute teachers to replace those attending PD sessions is common practice. However, the scale of PD being contemplated here would result in high substitute costs. As a rough estimate, a three-day initial mathematics workshop (the minimum we feel is needed to support a new primary/elementary curriculum), would require about 3,000 substitute teacher days in the first year of implementation, at a cost of around $600,000.[17]

[17] This estimate is based on an average hourly rate of $40 for substitute teachers on the “low rate” paid for an assignment of 3 days or less. The exact cost depends on the place on the salary scale of those hired as substitutes

Finally, the summer institute approach used in recent years seems to have been attractive and successful. However, there is some doubt if this approach can be scaled up to meet the demand. Although teacher demand has exceeded capacity, there is no assurance that all teachers needing the PD would attend summer sessions and there is no way that this could be required. The ten-month school year is so firmly entrenched in the teacher salary scale that it seems virtually impossible to conduct significant PD during the summer without providing some incentives for teachers to attend. Summer institutes could possibly be made fairly attractive by offering to pay teachers about the same amount as substitutes for the days they attend or on a scale comparable to that used for the public examinations marking board, also conducted during the summer. The cost would be no greater than the substitute option and would be less disruptive to instruction. Whether this would attract most of the teachers involved in implementation of a new program is unknown but could probably be investigated fairly easily.

A further issue in offering summer institutes would be finding sufficient resource people available at that time of year. Certainly the provincial mathematics consultant could not be expected to conduct all of the needed sessions. The numeracy support teachers would be well placed to do this work but it is not clear if they would be willing to take summer time for this purpose.

We understand that in the Western provinces, publishers have been persuaded to support PD work, in return for adoption of their texts. It might be possible to negotiate such an arrangement as part of a textbook adoption package. This could be done under any of the approaches discussed

As for timing within the school year, we believe that holding PD sessions late in the school year is a more viable approach than the current practice of conducting most of this work early in the year. To begin with, PD designed to support new curriculum needs to be held prior to teachers having to confront that curriculum. Holding sessions in the Fall of the year in which implementation is occurring is an invitation for teachers to start on the wrong foot before having a chance to become adequately acquainted with the program. A session held in June of the year before would make much more sense, and would have the added advantage of placing the new resources in the hands of teachers early enough to allow the teacher to follow up on the PD session.

If schools are to be closed for the PD sessions, a strong argument can also be made for doing this late in the school year. By many accounts, not much is done in the last few days of the school year in any case. In our view, closing schools for three days in late June would have much less impact on students than doing so in September or October. While an argument can be made that the end of the school year is a busy time for teachers, the same can be said for the beginning of the year.

One of the difficulties with the current model for PD delivery is that it is a zero sum game. It is impossible to scale up the level of PD in one area except at the expense of others. In making recommendations on mathematics, we have no desire to impinge on other areas. A way needs to be found to either expand the total PD resource pool or to change the PD delivery model, especially in the case of support of program implementation. This is where a school-based model becomes attractive.

Some years ago, the Government of Newfoundland and Labrador (1994) introduced a proposal to treat the school as the primary unit for program implementation and to enhance teacher leadership at the school level. This involved the designation of "lead teachers," recognized for their exceptional teaching skills and subject area expertise and given salary credit and release time to take on responsibilities for program implementation. The proposal was essentially that schools develop staffing policies that, over time, would ensure a balanced staff, with someone able to take a leadership role in all of the major subject areas. This proposal was lost in the face of the large scale structural reforms and financial constraints of the time. Nevertheless, our view is that the most promising way to change the program implementation model is to work towards developing leadership capacity at the school level.

The survey results suggest that there should be sufficient mathematics capacity at the intermediate and high school level to allow this to occur at these levels. Individuals designated as department heads should be well placed to take on this role, possibly with some support through release time[18], which should be explicitly linked to program implementation and support for other teachers.

[18] We understand that department heads receive stipends in recognition of their additional responsibilities, which we would suggest should include PD work with other teachers where needed.

However, this is not the case for primary/elementary mathematics, where relatively small numbers of teachers can be found with the level of mathematics background required to take a leadership role in this area. While our first preference would be to argue for redirecting the funds now being allocated to numeracy support teachers to support of school-based lead teachers, it seems unlikely that sufficient leadership capacity exists to make this happen in the short term. Most of the numeracy support teachers are assigned to the primary/elementary levels, which seems appropriate under current circumstances. Although the responsibilities of these teachers reach across several schools, they do have the scope to act as itinerant lead teachers at least until a more comprehensive lead teacher approach can be implemented.

The role of the numeracy support teachers is still evolving and there is no way to determine if this influx of specialized staff will have any impact. The work of these teachers needs to be closely monitored to determine if there is payoff for the substantial investment in personnel that this program entails. However, there is reasonable evidence that a school-based approach to program improvement is better than an approach based at the district or higher levels. It follows that the expertise available through the numeracy support teachers is most appropriately applied at the school level. Assignment of these teachers to clusters of schools might be necessary for small schools where it might prove to be impossible to recruit appropriately qualified persons. The question is whether this role is best exercised through direct PD work with teachers in their own schools or through more indirect support such working within the classroom either on a demonstration basis or to give teachers some relief for planning or PD work.

Our sense is that in the first years of implementation of the new mathematics program at the primary and elementary levels the numeracy support teachers would have to become the primary PD delivery staff. Among the timing options discussed, our view is that the intensive PD work should be done in June of each year of implementation, with the numeracy support teachers as the primary facilitators. These teachers would then follow up the initial sessions over the whole of the next school year, essentially acting as lead teachers, with responsibility extending over several schools.

Recommendation 6

That implementation of the proposed changes to the mathematics curriculum be accompanied by an introductory professional development program designed to introduce the curriculum to all mathematics teachers at the appropriate grade levels prior to the first year of implementation.

From a substantive perspective, there is no preferred way of implementing this PD program. However, the above discussion points to summer institutes as the approach to be first considered. Summer institutes do not interfere with the school schedule and they would be no more costly than use of substitute teachers. However, given the possibility of difficulties in staffing such institutes and of attracting all teachers, as well as in consideration of overall cost, we would argue that school closings near the end of the school year would be a workable approach for teachers at the primary and elementary levels. If cost is not an issue, substitute teachers could be used. It would be appropriate to canvass teachers on their own preferences, recognizing that there may not be a consensus and that it might not be feasible to implement more than one approach, because this would fragment the target population.

Recommendation 7

That at least partial support for professional development be negotiated with publishers as part of a textbook adoption package.

Recommendation 8

That numeracy support teachers have a primary role in delivery of PD for primary/elementary teachers.

Recommendation 9

That the responsibilities of mathematics department heads in intermediate and high schools (whether 7-12, 10-12 or any other combination) include facilitating introductory PD sessions and follow-up of these sessions.

Recommendation 10

That, in the short term, numeracy support teachers assigned to Grades K-6 be considered as lead teachers for mathematics in the schools for which they are responsible.

Recommendation 11

That the work of numeracy support teachers be systematically monitored for at least two years, using methods designed to assess their impact on fidelity of implementation and on outcomes.

Recommendation 12
That, following this period, a determination be made of whether this program should be continued or whether the resources would be better utilized to support lead teachers at the individual school level.

Instructional Practices

Most of what is known about instructional practices is not specific to mathematics but applies to classroom teaching in general. The main features of effective classrooms are reasonably well known, and may be summarized by reiterating points from the literature review chapter:

- Provide adequate time for learning
- Maximize teacher and student engagement during this time
- Ensure a positive disciplinary climate which avoids disruption and loss of time
- Engage in academically meaningful work at a moderate level of difficulty
- Have high expectations
- Maximize content coverage
- Monitor learning and use assessment to enhance learning

Two of the points, namely providing adequate time for learning and maximizing content coverage have been addressed indirectly in the discussion of curriculum, from the perspective of matching the expected content to the time available rather than increasing time on mathematics at the expense of other areas. While it might be argued that the current curriculum certainly maximizes content coverage, the prevailing view in the field is that this has been carried so far as to be counterproductive. Our sense is that teachers are striving to maximize content coverage, under conditions that many see as impossible. The revisions proposed are intended to bring content coverage into reasonable balance, allowing time to treat the content in sufficient depth to enhance learning.

Not much more needs to be said along these lines other than that every opportunity should be used to reinforce the features of effective teaching.

Recommendation 13

That the Department of Education reinforce, through reference in its curriculum documents and professional development activities, to well established features of effective teaching: maximizing the use of time, maximizing student engagement in academically meaningful work, high expectations, maximizing content coverage, monitoring and using assessment to improve learning.

As for mathematics itself, the literature review indicates that teaching for conceptual meaning and teaching for skill development are not contradictory. It is important to reiterate that the curriculum change that would take place under the recommended approach is not intended to tip the scale back to a purely procedural or algorithmic approach to mathematics. The WNCP 2006 and 2007 frameworks retain an approach derived from the NCTM *Principles and Standards* and is consistent with the priorities set out in the *Curriculum Focal Points* document, which shifts the balance in a way that ensures that important procedural skills are taught in the early grades.

A specific point related to instruction in mathematics is the place of homework. Large scale studies generally point to positive effects for homework. The 2001 SAIP mathematics assessment (CMEC, 2003) indicated that 13-year-old and 16-year-old students in this province spent more time on mathematics homework than those in most other Canadian jurisdictions. This was before the current mathematics program was implemented. Judging from the comments received in this study, the amount of homework is considered excessive by many parents and teachers. Parent complaints centered around the total amount of homework, the use of homework to compensate for lack of time to cover the program adequately and the inability of many parents to cope with the conceptual demands of the curriculum. It seems from what we heard that parents support practice exercises in conventional mathematical operations as homework but are uncomfortable with other types of work. Teachers were more concerned with a perception that many students are no longer inclined to do homework but also referenced the need to use homework to keep up with the necessarily fast pace of the program.

Somewhat related to homework is the issue of remediation. Assigning homework as a catch-up device is obviously not workable in all situations, and students most in need of help may be least likely to get this help at home. A few of our focus group participants noted that many students could benefit from a small amount of additional time and assistance to ensure that they do not fall too far behind the grade level expectations. There is no provision for such help under the current system for special education teacher allocations unless the student is diagnosed with some kind of disability. All that might be needed in some cases is some one-on-one or group time in addition to regular mathematics classes. In the absence of information on proposed changes to the current model for special education, we can only state the view that this should include clear provision for remediation in mathematics.

Recommendation 14

That the Department of Education reinforce the value of homework, establish guidelines on the amount and type of homework to be assigned, especially in the early grades, and develop and disseminate a parent guide to homework.

Recommendation 15

That any revisions to the model for providing services to special needs students include provision for remedial work for those students requiring additional time to meet grade level expectations.

REFERENCES

Bloom, B.S. (1976). *Human Characteristics and School Learning.* New York: McGraw-Hill.

Brophy, J.E. (1997). *Effective Instruction.* In H.J. Walberg & G.D. Haertel (Eds.), Psychology and Educational Practice. Berleley, CA: McCutchan

Brophy, J.E. & Good, T.L. (1986). *Teaching Behavior and Student Achievement*. In M.C Wittrock, (Ed.), Handbook of Research on Teaching, Third Edition. New York: Macmillan.

Bussière, P. Cartwright, F. & Knighton, T. (2004). *Measuring Up: the Performance of Canada's Youth in Mathematics, Reading and Science.* Ottawa: Statistics Canada

Bussière, P., Knighton, T. & Pennock, D. (2007). *Measuring Up: the Performance of Canada's Youth in Science, Reading and Mathematics: dirst Results for Canadians Aged 15.* Ottawa: Statistics Canada

Carroll, J. B. (1963). A Model for School Learning. *Teachers College Record,* 64, 8, 723-733.

Carroll, J. B. (1989). The Carroll Model: A 25-Year Retrospective and Prospective View. *Educational Researcher*, 18, 1, 26-31.

Cooper, H., Robinson, J.C., & Patall, E.A. (2006) Does Homework Improve Academic Acievement? A Synthesis of Research, 1987-2003. *Review of Educational Research*, 76,1, 1-62.

Council of Ministers of Education, Canada. (2003). *Mathematics Learning: the Canadian Context*. Toronto: Author.

Crocker, R.K. (1989) *Towards an Achieving Society: Report of the Task Force on Mathematics and Science Education*. St. John's: Government of Newfoundland and Labrador.

Dall'Alba, G. & Sandberg, J. (2006). Unveiling Professional Development: A Critical Review of Stage Models. *Review of Educational Research*, 76,3, 383-412.

Farran, S. (2007). *State of Unreadiness*. Maclean's. November 19, 2007.

Government of Newfoundland and Labrador. (1994). *Adjusting the Course, Part II: Improving the Conditions for Learning. .* St. John's: Author.

Government of Newfoundland and Labrador, Department of Education (2004). *Report from the Educational Staff Record Survey, 2002-03. .* St. John's: Author.

Government of Newfoundland and Labrador, Department of Education. Mathematics Curriculum Guides and other Support Documents for the Mathematics Curriculum , K-12. available from http://www.ed.gov.nl.ca/edu/sp/mathlist.htm).

Hiebert, J and Grouws, D, (2007). *The Effects of Classroom Mathematics Teaching on Students' Learning*. In Frank, K. Lester, Jr. Ed. Second Handbook of Research on Mathematics Teaching and Learning. Charlotte, NC: Information Age Publishing.

Hold Fast Consultants, 2004. WNCP Mathematics Research Project: Final Report. Victoria, B.C.: Author

Jacobs, V.R., Franke, M.L., Carpenter, T.P., Levi, L., & Battey, D. (2007).Professional Development Focusing on Children's Algebraic Reasoning in Elementary School. Journal for Research in Mathematics Teaching and Learning, 38,3,258-268.

Ma, L. (1999). *Knowing and Teaching Elementary Mathematics: Teachers' Understanding of Fundamental Mathematics in China and the United States*. Mahwah, NJ: Lawrence Erlbaum Associates.

Marshall, J. (2006). Math wars 2: It's the Teaching, Stupid!. Phi Delta Kappan, 87,5,356-363.

Marzano, R. (2003) *What Works in Schools: Translating Research into Action*. Washington: Association for Supervision and Curriculum Development

Miller, D & Glover, D. (2007). *Into the Unknown: The Professional*

Development Induction Experiences of Secondary Mathematics Teachers Using Interactive Whiteboard Technology. Learning, Media and technology, 32,3, 319-331.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics. (2000). *Principles and Standards for School Mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics. (2006). *Curriculum Focal Points* Reston, VA: Author.

Organisation for Economic Cooperation and Development. (2001). *Knowledge and Skills for Life: First Results from PISA, 2000.* Paris: Author.

Organisation for Economic Cooperation and Development. (2004). *Learning for Tomorrow's World: First Results from PISA, 2003.* Paris: Author.

Ross, J., Hogaboam-Gray A., & Bruce, C. (2006*). The Impact of a Professional Development Program on Student Achievement in Grade 6 Mathematics.* Journal of Mathematics Teacher Education, 9,6,551-577.

Schoenfeld, A. H (2003). *The Math Wars*. Educational Policy, 18,1,253-286.

Stigler, W. & Hiebert, J. (1999). *The Teaching Gap: Best Ideas for the World's Teachers for Improving education in the classroom*. New York: Free Press.

Scheerens, J. and Bosker, R. (1997). *The Foundations of Educational Effectiveness*. New York: Elsevier.

Wang, M. C. Haertel, G. D. and Walberg, H. J. (1993). Toward a knowledge base for school learning. *Review of Educational Research*. 63(3), 249-294.

Western and Northern Canadian Protocol (2006). *The common curriculum framework for K-9 mathematics.*
Retrieved from http://www.wncp.ca/math/ccfkto9.pdf.

Western and Northern Canadian Protocol (2007). *The common curriculum framework for K-9 mathematics.*
Retrieved from http://www.wncp.ca/math/ccfkto9.pdf.